CSC 165

lastBounds
virtual monday

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Test 2 — Next Wednesday
A2 - due on Friday!

key counterexample

Let $\mathcal{F} = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0}\}$, and consider the claim

$\forall f, g \in \mathcal{F}, f \in \mathcal{O}(g) \vee g \in \mathcal{O}(f)$

The claim is false, as the following counterexample shows:

$$f(n) = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases} \qquad g(n) = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge f(n) > cg(n)$

assume $c \in \mathbb{R}^+$, $B \in \mathbb{N}$. # generic choices

let $n = 2B$ Then $n \in \mathbb{N} \wedge n \geq B$ # $2B \geq B \wedge 2, B \in \mathbb{N}$ # $\mathbb{N}$ closed under $\times$

Then n is even # defn

Then $f(n) = 1 > 0 = cg(n)$ # $f(n) = 1$ # if n even # $g(n) = 0$ # n even.

Then $(\exists n \in \mathbb{N}, n \geq B, f(n) > cg(n))$

Then $\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N},$ ↓

$$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge f(n) > cg(n)$$

$\lim_{x\to\infty} \frac{x}{\log x} = \infty$

How about $n \notin \mathcal{O}(\log n)$?

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$

Statement: $\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, n \geq B \wedge \underbrace{n > c\log(n)}$

if $\lim_{x\to\infty} f(n) = \infty$ $\wedge \lim_{x\to\infty} g(n) = \infty$

Assume $c \in \mathbb{R}^+$, $B \in \mathbb{N}$ # generic $\wedge$ $\frac{n}{\log n} > c$

Then $\exists x \in \mathbb{R}^+, \forall y \in \mathbb{R}^+, y \geq x \Rightarrow \frac{y}{\log y} > c$

choose $x' \in \mathbb{R}^+, \forall y \in \mathbb{R}^+, y \geq x' \Rightarrow \frac{y}{\log y} > c$,

choose $n = \lceil x' \rceil + B + 1$ Then $n \in \mathbb{N} \wedge n \geq B \wedge n \geq x'$

Then $\frac{n}{\log n} > c$ # By the limit (L'H's rule).
& choice of n

Then $n > c\log n$ # multiply both sides
by $\log n$, and $n \geq 2$ by
construction

$2^{a+b} = 2^a 2^b$

Can you modify the last result to show that $2^n \notin \mathcal{O}(n^3)$?

You could transform it from $n \notin \mathcal{O}(3 \log n)$

Suppose I could show:

$\forall c \in \mathbb{R}^+, \forall B \in \mathbb{N}, \exists n \in \mathbb{N}, \boxed{n \geq B \wedge n > 3c \log n}$

like to then $2^n > 2^{3c \log n} = n^{3c}$

$n \geq B \quad \wedge$

$\boxed{n > \log c + 3 \log n}$

$\frac{n}{\log n} > \log c + 3$

$2^n > c n^3$

then $n > \boxed{\log c \cdot \log n} + 3 \log n$

works if $n \geq 2$

exponentiation

monotonic ↑

$\boxed{\lim_{x \to \infty} \frac{x}{\log x} = \infty}$

choose x s.t. $\frac{x}{\log x} > \boxed{\log c + 3}$

slide 5

test 2

coverage: proofs of the style of chapter 4 and 5 — up to end of today's lecture.

preparation: assignment 2, lecture slides, tutorial exercises

form: 50 minutes, no prove/disprove conu~~m~~ndrums

I owe you: today's annotated slides

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$$\lim_{x \to \infty^+} \frac{x}{\log x} = \infty \quad \boxed{\text{means}}$$

$$\forall z \in \mathbb{R}^+, \exists x \in \mathbb{R}^+, \forall y \in \mathbb{R}, y \geq x \Rightarrow \frac{y}{\log y} > z$$