

## free lunch

Work with 1 or 2 other students, and choose one of your group as the recorder. The recorder should keep a written record of his/her group's discussion of the problem below, and you may use this in writing an account of this problem-solving session. Use the following headings to organize the discussion:

- Understand the problem.
- Devise a plan.
- Carry out the plan.
- Look back.
- Acknowledge when, and how, you're stuck.

You are part of a group of friends who choose which to treat to lunch in the following manner:

1. They arrange themselves in an (approximate) circle.
2. They begin reciting the positive natural numbers, in order, in a counter-clockwise direction (viewed from above), starting with the friend at the northern extreme of the circle (who utters "one").
3. As a friend utters an even number, he or she is eliminated from the counting (and consideration for lunch). The counting "wraps around" so that those who avoided one of the dreaded even numbers on the first round may be exposed on subsequent rounds.
4. The last person left is treated to lunch by the others.

For example, if there are friends  $f_1$ ,  $f_2$ ,  $f_3$ ,  $f_4$ , and  $f_5$  arranged counter-clockwise, with  $f_1$  at the northern extreme, the first round would eliminate  $f_2$  and  $f_4$ . Then  $f_1$  and  $f_5$  would be eliminated in the next round, leaving  $f_3$  to enjoy the free lunch.

If there are  $n$  friends, where should you position yourself to get the free lunch? Do you have a technique that will work for any positive natural number  $n$ ? What would you do if the rules changed so that those who uttered numbers divisible by 3 were eliminated?

You can continue working on this problem at:

[https://wwwcgi.cdf.toronto.edu/~heap/cgi-bin/Solvent/wiki.pl?Problem\\_Solving\\_Home\\_Page/FreeLunch](https://wwwcgi.cdf.toronto.edu/~heap/cgi-bin/Solvent/wiki.pl?Problem_Solving_Home_Page/FreeLunch)

HINT 1, RECURSION: Is there any connection between the lunch number for  $n$  and for  $2n$ ?