CSC 165

program examples

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counting costs

want a coarse comparison of algorithms “speed” that ignores
hardware, programmer virtuosity

which speed do we care about: best, worst, average?
why?

define idealized “step” that doesn’t depend on particular hardware
and idealized “time” that counts the number of steps for a given input.

linear search

```
def LS(A,x) :
  """ Return index i such that x == L[i].  Otherwise, return -1 """
    i = 0
    while i < len(A) :
        if A[i] == x :
            return i
        i = i + 1
    return -1
```

Trace `LS([2,4,6,8],4)`, and count the time complexity $t_{\mathrm{LS}}([2,4,6,8],4)$

What is $t_{\mathrm{LS}}(A,x)$, if the first index where x is found is j?

What is $t_{\mathrm{LS}}(A,x)$ is x isn't in A at all?

worst case

denote the worst-case complexity for program P with input $x \in I$,
where the input size of x is n as
$W_P(n) = \max\{t_P(x) \mid x \in I \wedge \text{size}(x) = n\}$

The upper bound $W_P \in \mathcal{O}(U)$ means

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \;\Rightarrow\; \max\{t_P(x) \mid x \in I \wedge \text{size}(x) = n\} \leq cU(n)$$

That is: $$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall x \in I, \text{size}(x) \geq B \;\Rightarrow\; t_P(x) \leq cU(\text{size}(x))$$

The lower bound $W_P \in \Omega(L)$ means

$$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \;\Rightarrow\; \max\{t_P(x) \mid x \in I \wedge \text{size}(x) = n\} \geq cU(n)$$

That is: $$\exists c \in \mathbb{R}^+, \exists B \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq B \;\Rightarrow\; \exists x \in I, \text{size}(x) = n \wedge t_P(x) \geq cL(n)$$

bounding a sort

```
def IS(A) :
    """ IS(A) sorts the elements of A in non-decreasing order """
    i = 1
    while i < len(A) :
        t = A[i]
        j = i
        while j > 0 and A[j-1] > t :
            A[j] = A[j-1] # shift up
            j = j-1
        A[j] = t
        i = i+1
```

I want to prove that $W_{\text{IS}} \in \mathcal{O}(n^2)$.

scratch