CSC165 Fall 2009
Assignment 3

due Friday December 4th, 10pm

The aim of this assignment is to give you more practice with big-Oh proofs and algorithm complexity. Your solution should be submitted as a PDF file `a3.pdf` to the CDF submission site:

`https://www.cdf.toronto.edu/students/`

You are welcome to work with up to one other partner from this course on this assignment. You indicate a partnership by each partner submitting the file `partnerA3` to the CDF submission site. The content of the file should be the CDF userid of your partner.

1. Use the proof structure from this course to PROVE or DISPROVE the following claims. Please note that some of the claims are FALSE, and should be disproved.

   (a) Define $f(n) = 3n^3 + 5n$ and $g(n) = n^4 + 2n^2 + 7$. Claim: $f \in \mathcal{O}(g)$.

   (b) Use the same f and g from the previous question. Claim: $g \in \mathcal{O}(f)$.

   (c) Define $f(n) = 7^n + \ln(n)$ and $g(n) = n^3 + n$. Claim: $g \in \Omega(f)$.

   (d) Let $\mathcal{F} = \{f : \mathbb{N} \mapsto \mathbb{R}^{\geq 0}\}$. Define $\min(f, g)$ as $\forall n \in \mathbb{N}, \min(f, g)(n) = \min(f(n), g(n))$. Claim:

   $$\forall f, g, h \in \mathcal{F}, f \in \Omega(h) \wedge g \in \Omega(h) \Rightarrow \min(f, g) \in \Omega(h)$$

   (e) Does it work the other way? Define $\max(f, g)$ as $\forall n \in \mathbb{N}, \max(f, g)(n) = \max(f(n), g(n))$. Claim:

   $$\forall f, g, h \in \mathcal{F}, \max(f, g) \in \Omega(h) \Rightarrow f \in \Omega(h) \vee g \in \Omega(h)$$

2. Choose one of the four definitions of f below so that the worst-case running time of the algorithm DL is bounded above and below by the function you chose, that is $W_{\text{DL}} \in \mathcal{O}(f) \wedge W_{\text{DL}} \in \Omega(f)$. Prove that your bound is correct.

   $$f(n) = \log_2(n) \qquad f(n) = \sqrt{n} \qquad f(n) = n \qquad f(n) = n^2$$

```
def DL(n) :
   i = 0
   while 2**i < n : i += 1
   return i
```