CSC148 winter 2018

recursive structures
week 7

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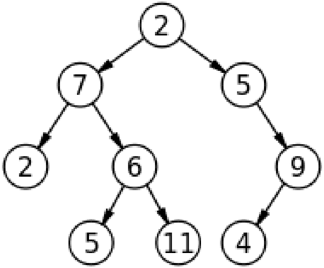
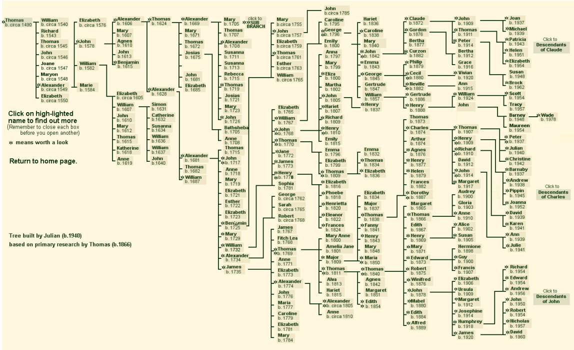
recursion, natural and otherwise



structure to organize information

patriarchal family tree...



terminology

- set of **nodes** (possibly with values or labels), with directed **edges** between some pairs of nodes
- One node is distinguished as **root**
- Each non-root node has exactly one parent.
- A **path** is a sequence of nodes $n_1, n_2, \ldots, n_k$, where there is an edge from n_i to n_{i+1}. The **length** of a path is the number of edges in it
- There is a unique path from the root to each node. In the case of the root itself this is just n_1, if the root is node n_1.
- There are no **cycles** — no paths that form loops.

more terminology

- **leaf**: node with no children
- **internal node**: node with one or more children
- **subtree**: tree formed by any tree node together with its descendants and the edges leading to them.
- **height**: 1 + the maximum path length from the root to some leaf.
- **depth**: length of a path from root to a node is the node's depth.
- **arity, branching factor**: maximum number of children for any node.

general tree implementation

```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the entire tree
    """
    def __init__(self, value: object=None,
                 children: List[Tree]=None) -> None:
        """
        Create Tree self with content value and 0 or more children
        """
        self.value = value
        # copy children if not None
        self.children = children[:] if children are not None else []
```

general form of recursion:

if ⟨condition to detect a base case⟩:

  ⟨do something without recursion⟩

else: # ⟨general case⟩

  ⟨do something that involves recursive call(s)⟩

how many leaves?

```
def leaf_count(t: Tree) -> int:
    """
    Return the number of leaves in Tree t.

    >>> t = Tree(7)
    >>> leaf_count(t)
    1
    >>> t = descendants_from_list(Tree(7),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> leaf_count(t)
    6
    """
```

height of this tree?

```
def height(t: Tree):
    """
    Return 1 + length of longest path of t.

    >>> t = Tree(13)
    >>> height(t)
    1
    >>> t = descendants_from_list(Tree(13),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> height(t)
    3
    """
    # 1 more edge than the maximum height of a child, except
    # what do we do if there are no children?
```

arity, or branching factor

```
def arity(t: Tree) -> int:
    """
    Return the maximum branching factor (arity) of Tree t.

    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
```

filesystem example

```
from os import scandir, path
from tree import Tree


def path_to_tree(path_name: str) -> Tree:
    """
    Return a Tree representing filesystem starting from pathname.
    """
    return Tree((path_name, [f.name for f in scandir(path_name)]),
                [path_to_tree(path.join(path_name, f.name))
                 for f in scandir(path_name)
                 if f.is_dir()])
```

pass in a function

```
def count_if(t: Tree, p: Callable[[object], bool]) -> int:
    """
    Return number of values in Tree t that satisfy predicate p(value).

    Assume predicate p is defined on t's values

    >>> def p(v): return v > 4
    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> count_if(t, p)
    4
    >>> def p(v): return v % 2 == 0
    >>> count_if(t, p)
    5
    """
```

list the leaves

```
def list_leaves(t: Tree) -> int:
    """
    Return list of values in leaves of t.

    >>> t = Tree(0)
    >>> list_leaves(t)
    [0]
    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> list_ = list_leaves(t)
    >>> list_.sort() # so list_ is predictable to compare
    >>> list_
    [3, 4, 5, 6, 7, 8]
    """
```

traversal

The functions and methods we have seen get information from every node of the tree — in some sense they traverse the tree.

Sometimes the **order** of processing tree nodes is important: do we process the root of the tree (and the root of each subtree...) **before** or **after** its children? Or, perhaps, we process along **levels** that are the same distance from the root?

pre-order visit

```
def preorder_visit(t: Tree, act: Callable[[Tree], Any]) -> None:
    """
    Visit each node of Tree t in preorder, and act on the nodes
    as they are visited.

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> preorder_visit(t, act)
    0
    1
    4
    5
    6
    2
    7
    3
    """
    act(t)
    for c in t.children:
        preorder_visit(c, act)
```

postorder

```
def postorder_visit(t: Tree, act: Callable[[Tree], Any) -> None:
    """
    Visit each node of t in postorder, and act on it when it is visited

    >>> t = descendants_from_list(Tree(0),
                                   [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> postorder_visit(t, act)
    4
    5
    6
    1
    7
    2
    3
    0
    """
```

levelorder

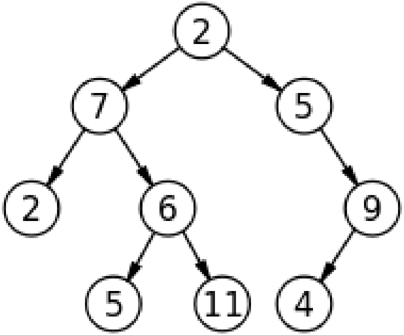
```
def levelorder_visit(t: Tree, act: Callable[[Tree], Any]) -> None:
    """
    Visit every node in Tree t in level order and act on the node
    as you visit it.

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> levelorder_visit(t, act)
    0
    1
    2
    3
    4
    5
    6
    7
    """
```

queues, stacks, recursion

You may have noticed in the last slide there were no recursive calls, and a **queue** was used to process a recursive structure in level order.

Careful use of a **stack** allows you to process a tree in preorder

preorder tracing...



notes...