CSC148-Section:L0301 Week#7-Friday

Instructed by

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Office hours: Wednesday 11-1, BA2230.

Slides adapted from Professor Danny Heap course material winter17

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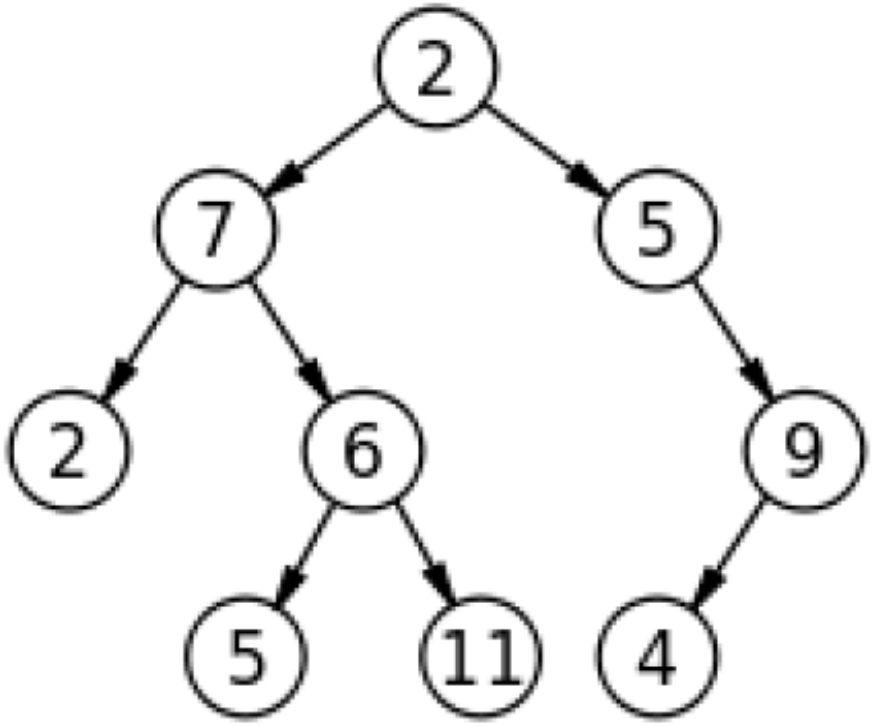
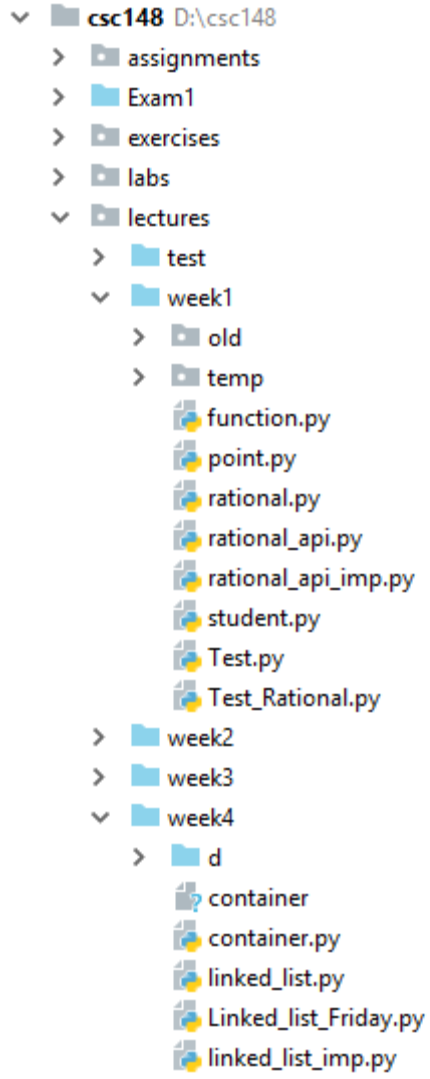
Outline

- Recursive structures
  - Trees

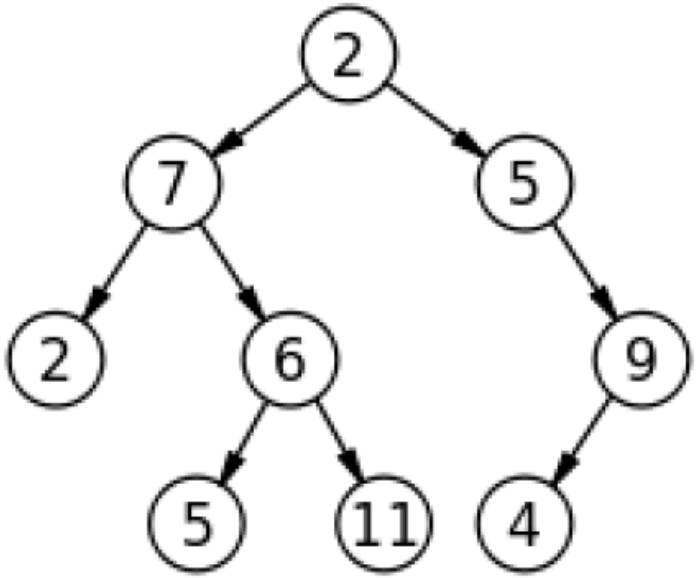
recursion, natural and otherwise



structure to organize information



Terminology



- **Tree** : set of **nodes** (possibly with values or labels), with directed edges between some pairs of nodes
- One node is distinguished as **root**
- Each non-root node has **exactly one parent.**
- A **path** is a sequence of nodes $n_1,n_2,\ldots,n_k$ where there is an edge from n_i,n_{i+1}
- The **length of a path** is the number of edges in it
- There is a unique path from the root to each node.
  - In the case of the root itself this is just n_1, if the root is node n_1.
- There are **no cycles -** no paths that form loops.

Terminology…

- **leaf:** node with no children.
- **internal node:** node with one or more children
- **subtree:** tree formed by any tree node together with its descendants and the edges leading to them.
- **height:** 1 + the maximum path length in a tree. A node also has a height, which is 1 + the maximum path length of the tree rooted at that node
- **depth:** length of a path from root to a node is the node's depth.
- **arity**, **branching factor**: maximum number of children for any node.

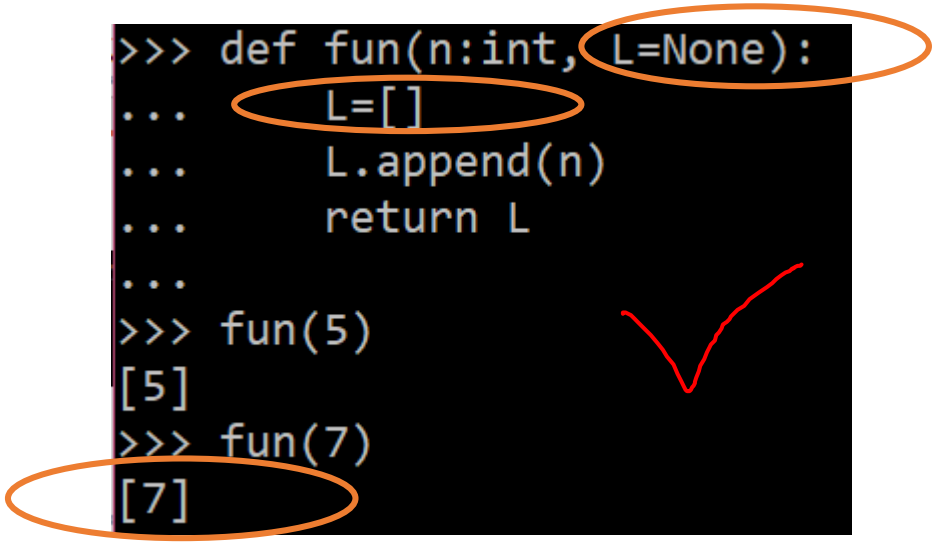
Tree ADT

Do Not assign a parameter to the empty list [] in a method definition instead make it None and assign it to [] inside the method body

```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the
entire tree.
    """
    def __init__(self, value: object=None, children:
                 List['Tree']=None) -> None:
        """
        Create Tree self with content value and 0 or more
          children
        """
        self.value = value
        # copy children if not None
        self.children = children.copy() if children else []
```

Do Not assign a parameter to the empty list [] in a method definition instead make it None and assign it to [] inside the method body

```
>>> def fun(n:int, L=[]):
...     L.append(n)
...     return L
...
>>> fun(5)
[5]
>>> fun(7)
[5, 7]
>>>
```



Template for Tree recursive functions

```
if t.children == [] :#Base case
    return  some_result
else:#General case
    return helper_function([your_rec_func(x) for x in t.children])
```

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how many leaves?

```
def leaf_count(t: Tree) -> int:
    """
    Return the number of leaves in Tree t.

    >>> t = Tree(7)
    >>> leaf_count(t)
    1
    >>> t = descendants_from_list(Tree(7),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> leaf_count(t)
    6
    """
```

```
def leaf_count(t: Tree) -> int:
    """
    Return the number of leaves in Tree t.
    >>> t = Tree(7)
    >>> leaf_count(t)
    1
    >>> t = descendants_from_list(Tree(7),[0, 1, 3, 5, 7, 9, 11,
    >>> leaf_count(t)
    6
    """
    if t.children==[]:
        return 1
    else:
        return sum([leaf_count(x) for x in t.children])
```

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height of this tree?

```
def height(t: Tree):
    """
    Return 1 + length of longest path of t.

    >>> t = Tree(13)
    >>> height(t)
    1
    >>> t = descendants_from_list(Tree(13),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> height(t)
    3
    """
    # 1 more edge than the maximum height of a child, except
    # what do we do if there are no children?
```

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```
def height(t: Tree) -> int:
    """
    Return 1 + length of longest path of t.
    >>> t = Tree(13)
    >>> height(t)
    1
    >>> t = descendants_from_list(Tree(13),[0, 1, 3, 5, 7, 9, 11,
    >>> height(t)
    3
    """
    # 1 more edge than the maximum height of a child, except
    # what do we do if there are no children?
    # helpful helper function
    if t.children ==[]:
        return 1
    else:
        return 1 + max([height(x) for x in t.children])
```

arity, or branching factor

```
def arity(t: Tree) -> int:
    """
    Return the maximum branching factor (arity) of Tree t.

    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
```

```
def arity(t: Tree) -> int:
    """
    Return the maximum branching factor (arity) of Tree t.
    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
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    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
    if t.children ==[]:
        return 0
    else:
        y= [arity(x) for x in t.children]
        return max(y) if max(y)>len(y) else len(y)
```

See next slide for another way of doing the same thing

```
def arity(t: Tree) -> int:
    """
    Return the maximum branching factor (arity) of Tree t.
    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
    if t.children ==[]:
        return 0
    else:
        return max(len(t.children),max([arity(x) for x in t.children]))
```