CSC148-Section:L0301 Week#6-Monday

Instructed by

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Office hours: Wednesday 11-1, BA2230.

Slides adapted from Professor Danny Heap course material winter17

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Outline

- Recursion
  - Quick review
  - Template
  - Implement recursive functions:
    - max_depth/rec_max/concat_strings/flatten/nested_contains of nested lists

Recursion

- A method calls itself

```
def factorial(num: int)->int:
    if num == 1:
        return 1
    else:
        return num * factorial(num-1)
```

Base case

General case

Recursion

- A **Recursive Function** has a **conditional structure** that consists of:
- Base case
  - species when/how to stop – has no recursion (does not call the function)
- General case
  - species **how to combine** **recursive subcalls**

Template for structural recursion

recursion when **input** is a recursive structure:

- Base case
  - if **input** cannot be decomposed into recursive sub-structures, you have a **base case** and you directly return a result ***without recursion***
- General case
  - if **input** can be decomposed into recursive sub-structures, solve them ***recursively*** and ***combine*** the result(s)

Template for structural recursion

This reduces your job to figuring out :

(a) how to detect whether the input can be decomposed or not.

(b) how what result to return for the base case, and

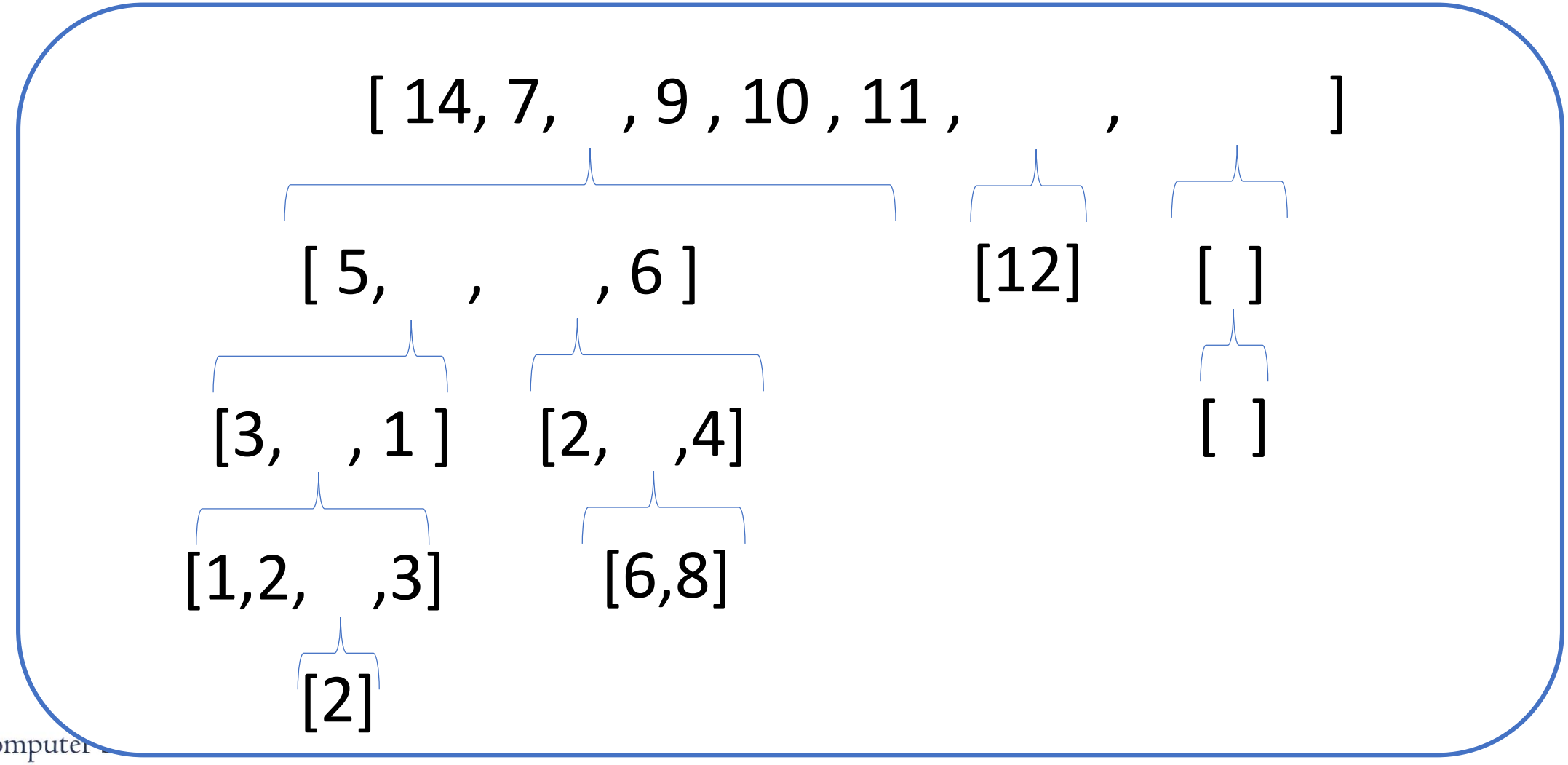
(c) which substructures to solve recursively and how to combine their solutions

Depth of list : `max_depth`
What is the depth of this list?

[14, 7, [5, [3, [1,2,[2],3], 1], [2,[6,8] ,4], 6], 9 , 10 , 11 , [12],[[]]]

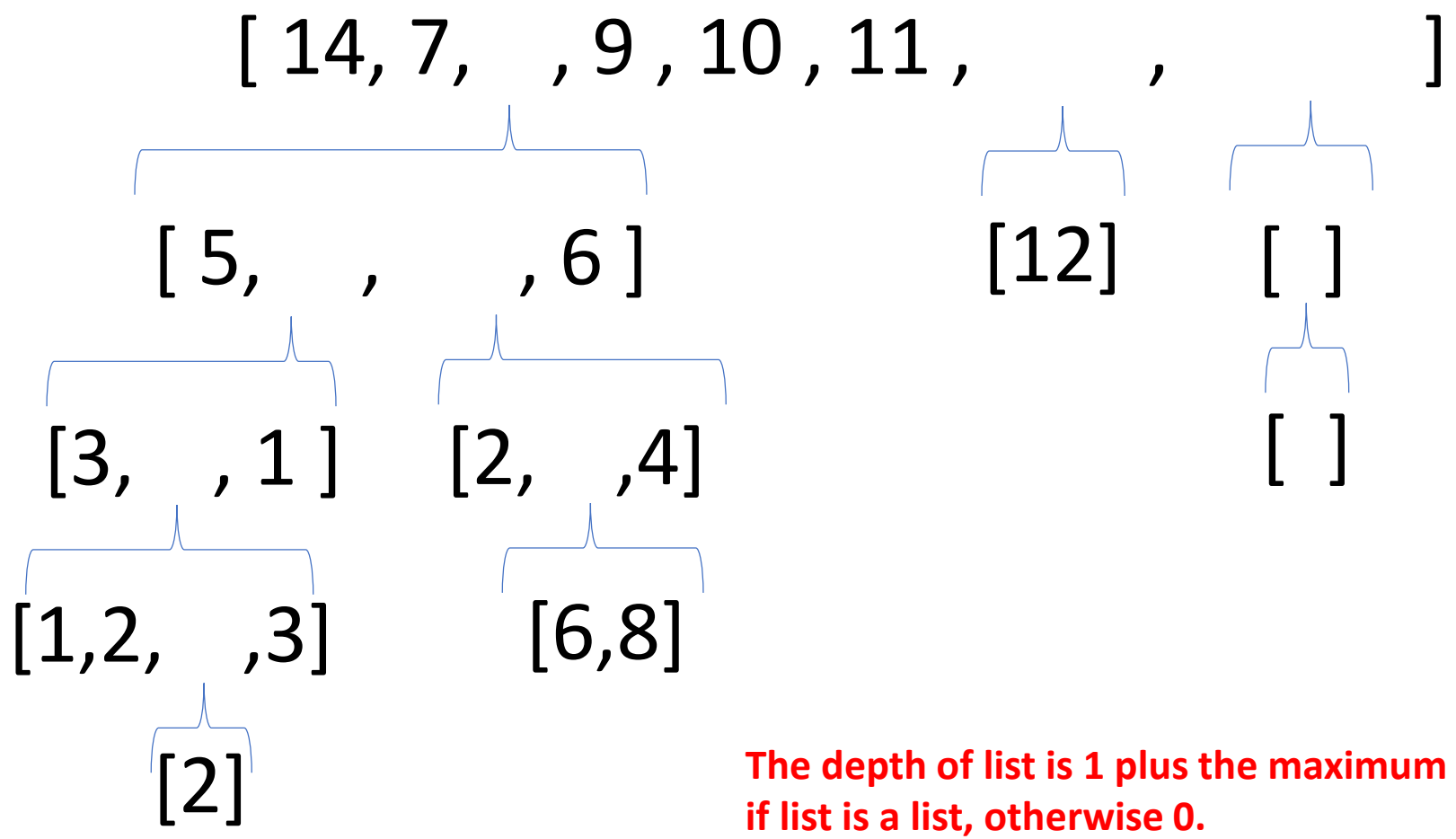
What is the depth of this list?

[14, 7, [5, [3, [1,2,[2],3], 1], [2,[6,8] ,4], 6], 9 , 10 , 11 , [12],[[]]]



What is the depth of this list?

[14, 7, [5, [3, [1,2,[2],3], 1], [2,[6,8] ,4], 6], 9 , 10 , 11 , [12],[[]]]



The depth of list is 1 plus the maximum depth of list 's elements if list is a list, otherwise 0.

What is the depth of this list?

- The depth of list is 1 plus the maximum depth of list 's elements if list is a list, otherwise 0

- The definition is **almost exactly the Python code you write**!

What is the depth of this list?

- The depth of list is 1 plus the maximum depth of list 's elements if list is a list, otherwise 0

- The definition is **almost exactly the Python code you write**!

```
if not isinstance(obj, list): # not list -part of Base case
    return 0
elif obj == []: # empty list - part of Base case
    return 1
else:  # General case
    return 1 + max([max_depth(x) for x in obj])
```

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Trace: `max_depth`

```
if not isinstance(obj, list):
    return 0
elif obj == []:
    return 1
else:
    return 1 + max([max_depth(x) for x in obj])
```

- Trace in increasing complexity;
- `Trace max_depth(17)`
- `Trace max_depth([])`
- `Trace max_depth([3, 17, 1])`
- `Trace max_depth([5, [3, 17, 1], [2, 4], 6])`
- `Trace max_depth([14, 7, [5, [3, 17, 1], [2, 4], 6], 9])`

at each step fill in values for recursive calls that have (basically) already been traced

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Trace: max_depth

```
if not isinstance(obj, list):
    return 0
elif obj == []:
    return 1
else:
    return 1 + max([max_depth(x) for x in obj])
```

- Trace max_depth(17)

-> 0

- Trace max_depth([])

-> 1

- Trace max_depth([3, 17, 1])

-> 1 + max([max_depth(3), max_depth(17), max_depth(1)])

-> 1 + max([0,0,0])

-> 1

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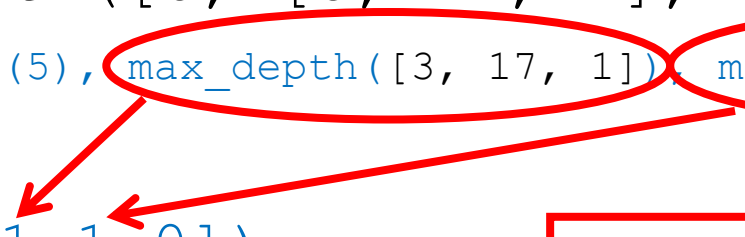
Trace: max_depth

```
if not isinstance(obj, list):
    return 0
elif obj == []:
    return 1
else:
    return 1 + max([max_depth(x) for x in obj])
```

- Trace max_depth([5, [3, 17, 1], [2, 4], 6])

```
-> 1 + max([max_depth(5), max_depth([3, 17, 1]), max_depth([2, 4]),max_depth(6)])
```



```
-> 1 + max([0,1,1,0])
-> 1 + 1
-> 2
```

When tracing on paper, at each step fill in values for recursive calls that have (basically) **already been traced**

Trace: max_depth

```
if not isinstance(obj, list):
    return 0
elif obj == []:
    return 1
else:
    return 1 + max([max_depth(x) for x in obj])
```

- Trace max_depth([14, 7, [5, [3, 17, 1], [2, 4], 6], 9])

```
-> 1 + max([max_depth(14), max_depth(7), max_depth([5, [3, 17, 1], [2, 4], 6]),max_depth(9)])
-> 1 + max([0,0,2,0])
-> 1 + 2
-> 3
```

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```
def max_depth(obj: object) -> int:
    """
    Return 1 + the maximum depth of obj's elements if obj is a list.
    Otherwise return 0.
    >>> max_depth(17)
    0
    >>> max_depth([])
    1
    >>> max_depth([1, "two", 3])
    1
    >>> max_depth([1, ["two", 3], 4])
    2
    >>> max_depth([1, [2, ["three", 4], 5], 6])
    3
    """
    if obj == []:
        return 1
    elif isinstance(obj,list):
        return 1+max([max_depth(x) for x in obj])
    else:
        return 0
```

Maximum number in nested list: `rec_max`

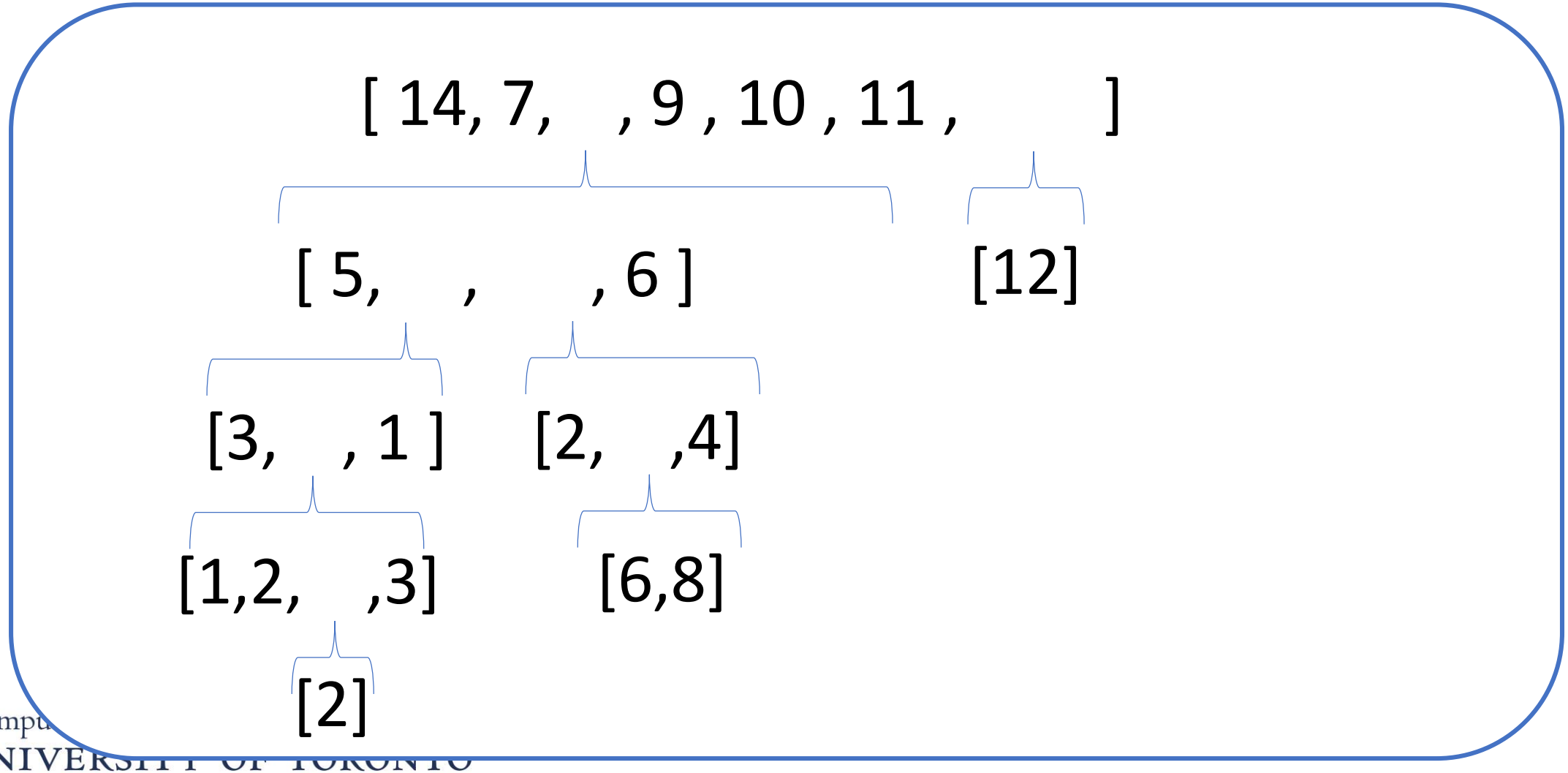
What is the max number in this list?

[14, 7, [5, [3, [1,2,[2],3], 1], [2,[6,8] ,4], 6], 9 , 10 , 11 , [12]]

Maximum number in nested list

What is the max number in this list?

[14, 7, [5, [3, [1,2,[2],3], 1], [2,[6,8] ,4], 6], 9 , 10 , 11 , [12]]



Maximum number in nested list: `rec_max`

- how would you find the max of non-nested list?

```
max(...)
```

- how would you build that list using a comprehension?

```
max([x for x in list])
```

- what should you do with list items that **were themselves lists**?

```
max([rec_max(x) for x in list_])
```

- get some intuition by tracing through:
  - flat lists,
  - lists nested one deep,
  - then two deep. . .

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Maximum number in nested list: `rec_max`

```
if isinstance(list_, list):
    return max([rec_max(x) for x in list_])
else:
    return list_
```

Trace: rec_max

```
if isinstance(list_, list):
    return max([rec_max(x) for x in list_])
else:
    return list_
```

- Trace in increasing complexity;

at each step fill in values for recursive calls that have (basically) already been traced

- `Trace rec_max([3, 5, 1, 3, 4, 7])`
- `Trace  rec_max([4, 2, [3, 5, 1, 3, 4, 7], 8])`
- `Trace  rec_max([6, [4, 2, [3, 5, 1, 3, 4, 7], 8], 5])`

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Trace : rec_max

```
if isinstance(list_, list):
    return max([rec_max(x) for x in list_])
else:
    return list_
```

- Trace in increasing complexity;

at each step fill in values for recursive calls that have (basically) already been traced

- Trace rec_max([3, 5, 1, 3, 4, 7])

-> max([rec_max(3), rec_max(5), rec_max(1), rec_max(3), rec_max(4), rec_max(7)])

-> max([3,5,1,3,4,7])

-> 7

- Trace rec_max([4, 2, [3, 5, 1, 3, 4, 7], 8])

-> max([rec_max(4), rec_max(2), rec_max([3, 5, 1, 3, 4, 7]), rec_max(8)])

-> max([4,2,7,8])

-> 8

When tracing on paper, at each step fill in values for recursive calls that have (basically) **already been traced**

Trace : rec_max

```
if isinstance(list_, list):
    return max([rec_max(x) for x in list_])
else:
    return list_
```

- Trace in increasing complexity;

at each step fill in values for recursive calls that have (basically) already been traced

- Trace rec_max([6, [4, 2, [3, 5, 1, 3, 4, 7], 8], 5])

```
-> max([rec_max(6), rec_max([4, 2, [3, 5, 1, 3, 4, 7], 8]), rec_max(5)])

-> max([6,8,5])
-> 8
```

When tracing on paper, at each step fill in values for recursive calls that have (basically) **already been traced**

```
def rec_max(list_: Union[List,int]) -> int:
    """
    Return the maximum int in nested lists

    lists and sublists must Not be empty or None
    >>> rec_max(17)
    17
    >>> rec_max([-1,0])
    0
    >>> rec_max([1, 2, 3])
    3
    >>> rec_max([1, [5, 3], 4])
    5
    >>> rec_max([1, [2, [9, 4], 5], 6])
    9
    """
    if isinstance(list_,list):
        return max([rec_max(x) for x in list_])
    else:
        return list_
```

Concatenating strings in nested list:

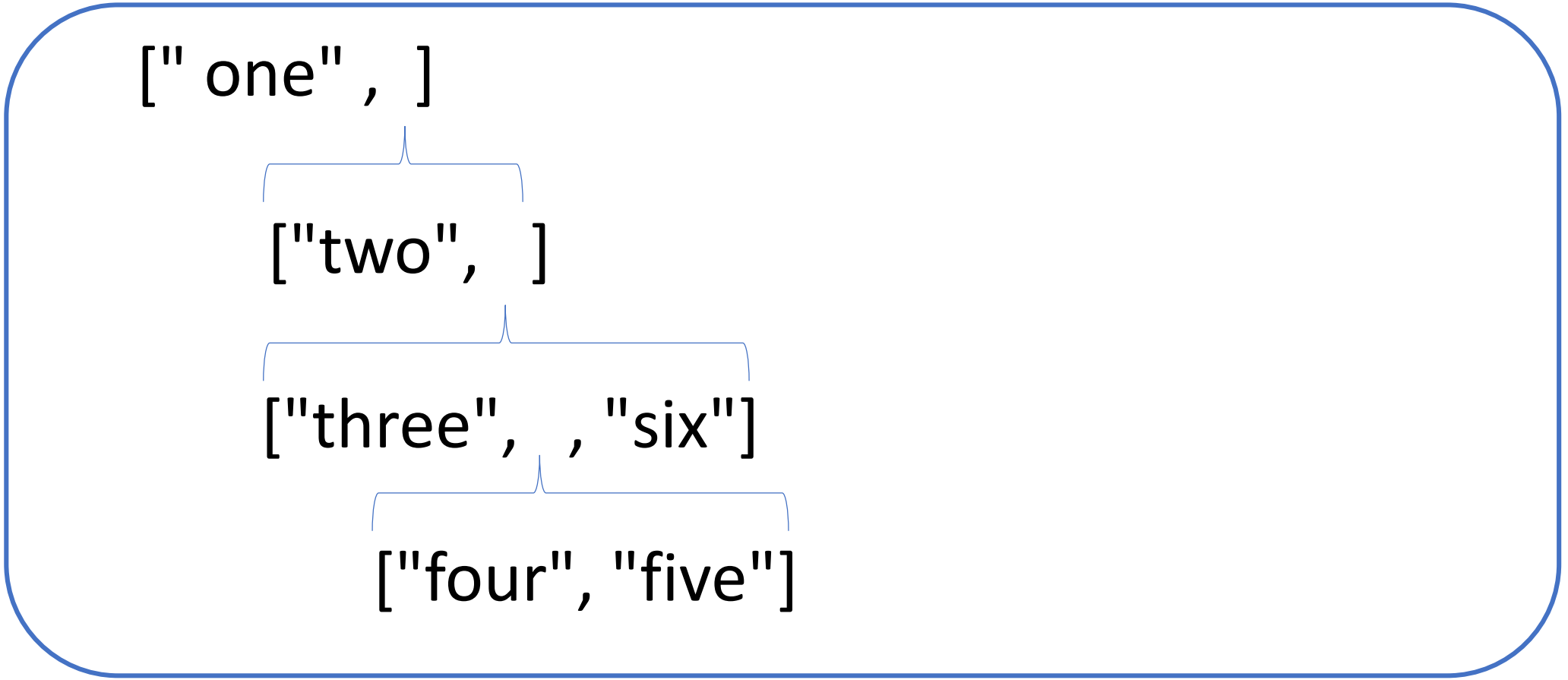
`concat_strings`

["one", ["two", ["three", ["four", "five"], "six"]]]

Concatenating strings in nested list:

```
concat_strings
```

["one" ,["two", ["three", ["four", "five"], "six"]]]



Concatenating strings in nested list: concat_strings

["one" ["two", ["three", ["four", "five"], "six"]]]

```
if isinstance(string_list, list): # General Case
    return "".join([concat_strings(x) for x in string_list])
else: # Base Case
    return string_list
```

Trace: concat_strings

```
if isinstance(string_list, list): # General Case
    return "".join([concat_strings(x) for x in string_list])
else: # Base Case
    return string_list
```

- Trace in increasing complexity;

at each step fill in values for recursive calls that have (basically) already been traced

- `Trace concat_strings(["now", "brown"])`

```
-> "".join([concat_strings("now"), concat_strings("brown")])
```

- `Trace` concat_strings(["how"])

- `Trace` concat_strings([])

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Trace:concat_strings

```
if isinstance(string_list, list): # General Case
    return "".join([concat_strings(x) for x in string_list])
else: # Base Case
    return string_list
```

- Trace in increasing complexity;

at each step fill in values for recursive calls that have (basically) already been traced

- `Trace concat_strings(["how", ["now", "brown"], "cow"])`

`-> "".join([concat_strings("now"), concat_strings("brown")])`

- `Trace concat_strings(["how", ["now", ["brown", "cow"]]])`

- `Trace concat_strings(["one" ["two", ["three", ["four", "five"], "six"]]])`

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```
def concat_strings(string_list):
    """
    Concatenate all the strings in possibly-nested string_list.
    @param list[str]|str string_list:
    @rtype: str
    >>> list_ = ["how", ["now", "brown1"], "cow1"]
    >>> concat_strings(list_)
    'hownowbrown1cow1'
    """
    return "".join([concat_strings(x) if isinstance(x,list)
                    else x
                    for x in string_list])
    # if isinstance(string_list,list):
    #     return "".join([concat_strings(x) for x in string_list])
    # else:
    #     return string_list
```

flatten

```
def flatten(list_: List) -> List:
    """
    Return a flattened list of nested lists

    >>> flatten([1, [5, 3], 4])
    [1, 5, 3, 4]
    """
```

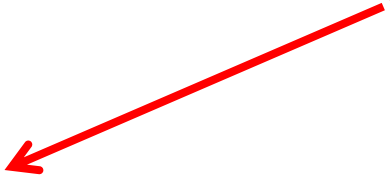
flatten

```
def flatten(list_: List) -> List:
    """
    Return a flattened list of nested lists

    >>> flatten([1, [5, 3], 4])
    [1, 5, 3, 4]
    """
    if isinstance(list_,list):
        return sum([flatten(x) for x in list_],[])
    else:
        return [list_]
```

flatten : using functional if

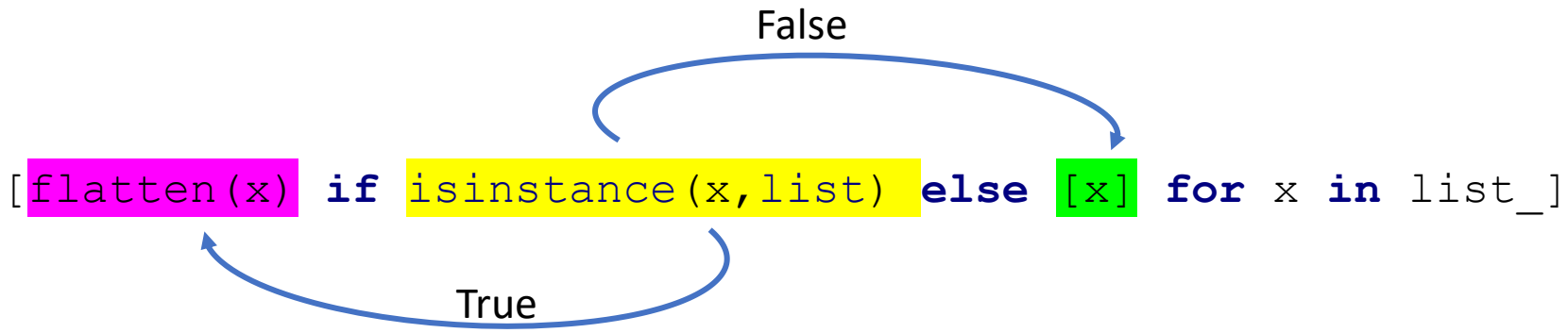
```
def flatten(list_: List) -> List:
    """
    Return a flattened list of nested lists

    >>> flatten([1, [5, 3], 4])
    [1, 5, 3, 4]
    """
    return sum([flatten(x) if isinstance(x,list)
                else [x]
                for x in list_],[])
```

using functional if



functional if

<expression 1> if <condition> else <expression 2>



- expression 1 is evaluated if condition is True
- expression 2 is evaluated if condition is False

```
def flatten(list_: List) -> List:
    """
    Return a flattened list of nested lists

    >>> flatten([1, [5, 3], 4])
    [1, 5, 3, 4]
    """
    # return sum([flatten(x) if isinstance(x,list)
    #             else [x]
    #             for x in list_],[])
    if isinstance(list_,list):
        return sum([flatten(x) for x in list_],[])
    else:
        return [list_]
```

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nested_contains

```
def nested_contains(list_: list, value: object) -> bool:
    """
    Return whether list_, or any nested sub-list of list_ contai
    >>> list_ = ["how", ["now", "brown"], 1]
    >>> nested_contains(list_, "now")
    True
    >>> nested_contains([], 5)
    False
    >>> nested_contains([5], 5)
    True
    """
    # check out Python built-in any
```

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nested_contains

```
def nested_contains(list_: list, value: object) -> bool:

    if isinstance(list_,list):
        return any([nested_contains(x,value) for x in list_])
    else:
        return value == list_
```

```
def nested_contains(list_: list, value: object) -> bool:
    """
    Return whether list_, or any nested sub-list of list_ contains value.
    >>> list_ = ["how", "now", "brown", 1]
    >>> nested_contains(list_, 1)
    True
    >>> nested_contains(list_, 3)
    False
    >>> list_ = ["how", ["now", "brown"], 1]
    >>> nested_contains(list_, "now")
    True
    >>> nested_contains([], 5)
    False
    >>> nested_contains([5], 5)
    True
    """
    # check out Python built-in any
    if isinstance(list_,list):
        return any([nested_contains(x,value) for x in list_])
    else:
        return value == list_
```