CSC148 winter 2017

recursive structures
week 7

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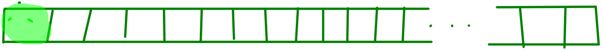
something linked lists do better than lists?

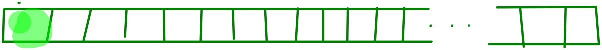


-> -> -> -> ... move all elements one position up

add

remove

<- <- <- ... move all elements down one position

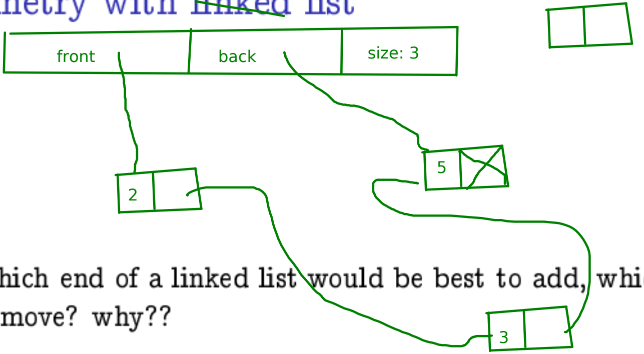


remove

add

list-based Queue has a problem: adding or removing will be slow.

symmetry with ~~linked~~ list



which end of a linked list would be best to add, which to remove? why??

build pop_front

... already have append

revisit Queue API

use an underlying LinkedList

revisit Stack API while we're at it

also use an underlying LinkedList

they're all Containers

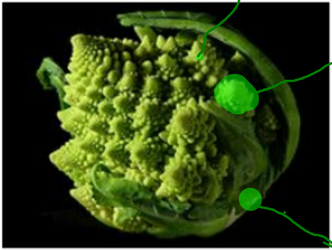
stress drive them through container_cycle, in container_timer.py:

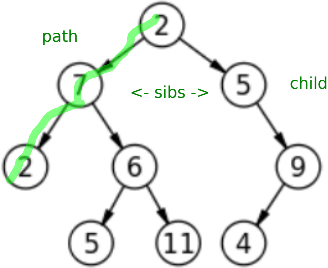
- list-based Queue
- linked-list-based Queue
- list-based Stack
- linked-list-based Stack

what matters is the growth rate

as Queue grows in size, list-based-Queue bogs down, becomes impossibly slow

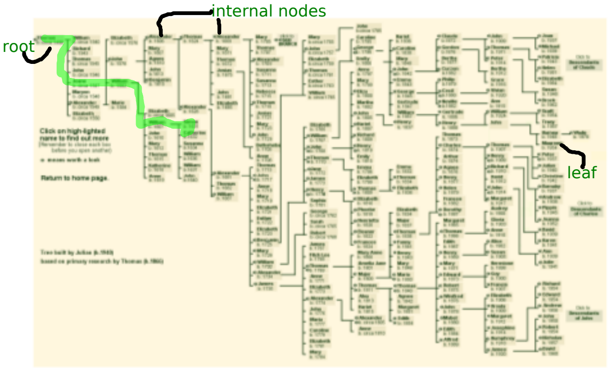
recursion, natural and otherwise





structure to organize information

patriarchal family tree...



terminology

- set of **nodes** (possibly with values or labels), with directed **edges** between some pairs of nodes

  arrows

- One node is distinguished as **root** at top (usually), no incoming arrows

- Each non-root node has exactly one parent. non-biological

- A **path** is a sequence of nodes $n_1, n_2, \ldots, n_k$, where there is an edge from n_i to n_{i+1}. The **length** of a path is the number of edges in it

- There is a unique path from the root to each node. In the case of the root itself this is just n_1, if the root is node n_1.

- There are no **cycles** — no paths that form loops.

more terminology

- **leaf**: node with no children
- **internal node**: node with one or more children
- **subtree**: tree formed by any tree node together with its descendants and the edges leading to them.
- **height**: 1 + the maximum path length in a tree. A node also has a height, which is 1 + the maximum path length of the tree rooted at that node
- **depth**: length of a path from root to a node is the node's depth.
- **arity, branching factor**: maximum number of children for any node.

general tree implementation

```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the entire tree
    """
    def __init__(self, value=None, children=None):
        """
        Create Tree self with content value and 0 or more children

        @param Tree self: this tree
        @param object value: value contained in this tree
        @param list[Tree] children: possibly-empty list of children
        @rtype: None
        """
        self.value = value
        # copy children if not None
        self.children = children.copy() if children else []
```

general form of recursion:

if ⟨condition to detect a base case⟩:

    ⟨do something without recursion⟩

else: # ⟨general case⟩

    ⟨do something that involves recursive call(s)⟩

how many leaves?

```
def leaf_count(t):
    """
    Return the number of leaves in Tree t.

    @param Tree t: tree to count number of leaves of
    @rtype: int

    >>> t = Tree(7)
    >>> leaf_count(t)
    1
    >>> t = descendants_from_list(Tree(7),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> leaf_count(t)
    6
    """
```

height of this tree?

```
def height(t):
    """
    Return 1 + length of longest path of t.

    @param Tree t: tree to find height of
    @rtype: int

    >>> t = Tree(13)
    >>> height(t)
    1
    >>> t = descendants_from_list(Tree(13),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> height(t)
    3
    """
    # 1 more edge than the maximum height of a child, except
    # what do we do if there are no children?
```

arity, or branching factor

```
def arity(t):
    """
    Return the maximum branching factor (arity) of Tree t.

    @param Tree t: tree to find the arity of
    @rtype: int

    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
```

pass in a function

```
def list_if(t, p):
    """
    Return a list of values in Tree t that satisfy predicate p(value).

    Assume predicate p is defined on t's values

    @param Tree t: tree to list values that satisfy predicate p
    @param (object)->bool p: predicate to check values with
    @rtype: list[object]

    >>> def p(v): return v > 4
    >>> t = descendants_from_list(Tree(0),
                                   [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> list_ = list_if(t, p)
    >>> list_.sort()
    >>> list_
    [5, 6, 7, 8]
    >>> def p(v): return v % 2 == 0
    >>> list_ = list_if(t, p)
    >>> list_.sort()
    >>> list_
```

list the leaves

```
def list_leaves(t):
    """
    Return list of values in leaves of t.

    @param Tree t: tree to list leaf values of
    @rtype: list[object]

    >>> t = Tree(0)
    >>> list_leaves(t)
    [0]
    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> list_ = list_leaves(t)
    >>> list_.sort() # so list_ is predictable to compare
    >>> list_
    [3, 4, 5, 6, 7, 8]
    """
```

traversal

The functions and methods we have seen get information from every node of the tree — in some sense they traverse the tree.

Sometimes the **order** of processing tree nodes is important: do we process the root of the tree (and the root of each subtree...) **before** or **after** its children? Or, perhaps, we process along **levels** that are the same distance from the root?

pre-order visit

```
def preorder_visit(t, act):
    """
    Visit each node of Tree t in preorder, and act on the nodes
    as they are visited.

    @param Tree t: tree to visit in preorder
    @param (Tree)->Any act: function to carry out on visited Tree node
    @rtype: None

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> preorder_visit(t, act)
    0
    1
    4
    5
    6
    2
    7
    3
```

postorder

```
def postorder_visit(t, act):
    """
    Visit each node of t in postorder, and act on it when it is visited

    @param Tree t: tree to be visited in postorder
    @param (Tree)->Any act: function to do to each node
    @rtype: None

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> postorder_visit(t, act)
    4
    5
    6
    1
    7
    2
    3
    0
    """
```

levelorder

```
def levelorder_visit(t, act):
    """
    Visit every node in Tree t in level order and act on the node
    as you visit it.

    @param Tree t: tree to visit in level order
    @param (Tree)->Any act: function to execute during visit

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> levelorder_visit(t, act)
    0
    1
    2
    3
    4
    5
    6
    7
    """
```

queues, stacks, recursion

You may have noticed in the last slide there were no recursive calls, and a **queue** was used to process a recursive structure in level order.

Careful use of a **stack** allows you to process a tree in preorder or postorder.

notes