CSC148 winter 2016

recursive structures
week 7

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recursion, natural and otherwise

tree structure

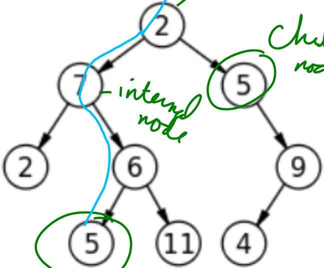
grandchild

child

root

path

root



child node

internal node

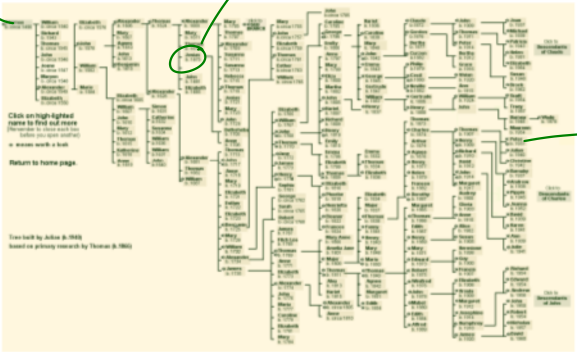
leaf

structure to organize information

patriarchal family tree...

root

internal node



leaf

terminology

- set of **nodes** (possibly with values or labels), with directed **edges** between some pairs of nodes
- One node is distinguished as **root**
- Each non-root node has exactly one parent. (unlike human family)
- A **path** is a sequence of nodes $n_1, n_2, \ldots, n_k$, where there is an edge from n_i to n_{i+1}. The **length** of a path is the number of edges in it length of path in single node is 0
- There is a unique path from the root to each node. In the case of the root itself this is just n_1, if the root is node n_1.
  path of length 0
- There are no **cycles** — no paths that form loops.

more terminology

- **leaf**: node with no children

Every node is one of these 2

- **internal node**: node with one or more children
- **subtree**: tree formed by any tree node together with its descendants and the edges leading to them.

tree made up of subtrees

- **height**: 1 + the maximum path length in a tree. A node also has a height, which is 1 + the maximum path length of the tree rooted at that node — There is another definition you may see which has the height as 1 less
- **depth**: Height of the entire tree minus the height of a node is the depth of the node.


arity = 3

- **arity, branching factor**: maximum number of children for any node.

general tree implementation

```
class Tree:
    """
    A bare-bones Tree ADT that identifies the root with the entire tree
    """
    def __init__(self, value=None, children=None):
        """
        Create Tree self with content value and 0 or more children

        @param Tree self: this tree
        @param object value: value contained in this tree
        @param list[Tree] children: possibly-empty list of children
        @rtype: None
        """
        self.value = value
        # copy children if not None
        self.children = children.copy() if children else []
```

No way to represent an empty tree here

general form of recursion:

if ⟨condition to detect a base case⟩:

base case cannot be reduced to recursive cases

⟨do something without recursion⟩

else: # ⟨general case⟩

⟨do something that involves recursive call(s)⟩

how many leaves?

```
def leaf_count(t):
    """
    Return the number of leaves in Tree t.

    @param Tree t: tree to count number of leaves of
    @rtype: int

    >>> t = Tree(7)
    >>> leaf_count(t)
    1
    >>> t = descendants_from_list(Tree(7),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> leaf_count(t)
    6
    """
```

if t is a leaf → 1
otherwise → sum the leaves in t's children

height of this tree?

```
def height(t):
    """
    Return 1 + length of longest path of t.

    @param Tree t: tree to find height of
    @rtype: int

    >>> t = Tree(13)
    >>> height(t)
    1
    >>> t = descendants_from_list(Tree(13),
                                  [0, 1, 3, 5, 7, 9, 11, 13], 3)
    >>> height(t)
    3
    """
    # 1 more edge than the maximum height of a child, except
    # what do we do if there are no children?
```

if t is a leaf → 1
otherwise 1 + max of children's heights

arity, or branching factor

```
def arity(t):
    """
    Return the maximum branching factor (arity) of Tree t.

    @param Tree t: tree to find the arity of
    @rtype: int

    >>> t = Tree(23)
    >>> arity(t)
    0
    >>> tn2 = Tree(2, [Tree(4), Tree(4.5), Tree(5), Tree(5.75)])
    >>> tn3 = Tree(3, [Tree(6), Tree(7)])
    >>> tn1 = Tree(1, [tn2, tn3])
    >>> arity(tn1)
    4
    """
```

leaf → 0
otherwise → max(#children, children's arities)

pass in a function

```
def list_if(t, p):
    """
    Return a list of values in Tree t that satisfy predicate p(value).

    Assume predicate p is defined on t's values

    @param Tree t: tree to list values that satisfy predicate p
    @param (object)->bool p: predicate to check values with
    @rtype: list[object]

    >>> def p(v): return v > 4
    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> list_ = list_if(t, p)
    >>> list_.sort()
    >>> list_
    [5, 6, 7, 8]
    >>> def p(v): return v % 2 == 0
    >>> list_ = list_if(t, p)
    >>> list_.sort()
    >>> list_
```

leaf → [t.value] if p(t.value) else []

otherwise → ↳ + gather_list([list_if(c, p) for c in t.children])

list the leaves

```
def list_leaves(t):
    """
    Return list of values in leaves of t.

    @param Tree t: tree to list leaf values of
    @rtype: list[object]

    >>> t = Tree(0)
    >>> list_leaves(t)
    [0]
    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7, 8], 3)
    >>> list_ = list_leaves(t)
    >>> list_.sort() # so list_ is predictable to compare
    >>> list_
    [3, 4, 5, 6, 7, 8]
    """
```

leaf → [t.value]
otherwise → gather_lists([list_leaves(c) for c in t.children])

traversal

The functions and methods we have seen get information from every node of the tree — in some sense they traverse the tree.

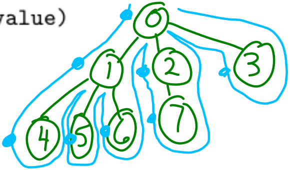
Sometimes the **order** of processing tree nodes is important: do we process the root of the tree (and the root of each subtree...) **before** or **after** its children? Or, perhaps, we process along **levels** that are the same distance from the root?

pre-order visit

```
def preorder_visit(t, act):
    """
    Visit each node of Tree t in preorder, and act on the nodes
    as they are visited.

    @param Tree t: tree to visit in preorder
    @param (Tree)->Any act: function to carry out on visited Tree node
    @rtype: None

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> preorder_visit(t, act)
    0
    1
    4
    5
    6
    2
    7
    3
```

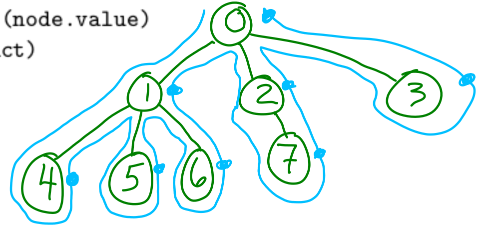


postorder

```
def postorder_visit(t, act):
    """
    Visit each node of t in postorder, and act on it when it is visited

    @param Tree t: tree to be visited in postorder
    @param (Tree)->Any act: function to do to each node
    @rtype: None

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> postorder_visit(t, act)
    4
    5
    6
    1
    7
    2
    3
    0
    """
```

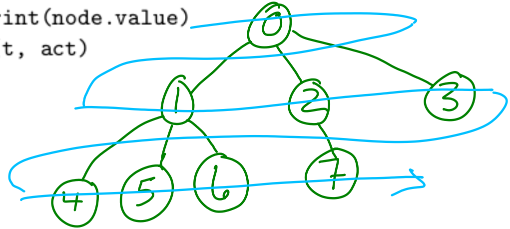


levelorder

```
def levelorder_visit(t, act):
    """
    Visit every node in Tree t in level order and act on the node
    as you visit it.

    @param Tree t: tree to visit in level order
    @param (Tree)->Any act: function to execute during visit

    >>> t = descendants_from_list(Tree(0),
                                  [1, 2, 3, 4, 5, 6, 7], 3)
    >>> def act(node): print(node.value)
    >>> levelorder_visit(t, act)
    0
    1
    2
    3
    4
    5
    6
    7
    """
```



queues, stacks, recursion

~~code~~
corresponding
to ...

You may have noticed in the last slide there were no recursive calls, and a **queue** was used to process a recursive structure in level order.

Careful use of a **stack** allows you to process a tree in preorder or postorder.

queues, stacks, recursion

You may have noticed in the last slide there were no recursive calls, and a **queue** was used to process a recursive structure in level order.

Careful use of a **stack** allows you to process a tree in preorder or postorder.