CSC148 winter 2016

reading recursion

week 6

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February 26, 2016

Outline

test #1 follow-up

recursion on nested lists

recursion with turtles

announcements

- office hours, Monday/Wednesday/Thursday 3–5, BA4270 OR BA2230
- also CS help centre Wednesday and Thursday 4–6

length, inter-section comparison

- mean of the 10 a.m. and 1 p.m. was a statistical tie, 6 p.m. was lower
- 6 p.m. mean is adjusted 0.5/30 higher
- overall average 67.9%
- there was a **lot** of writing, see the proposal several slides on

post-test exercise

- 1% post-test exercise, follow instructions on sticker, either on last page of test paper, or an inner page if there is no room
- exercise is due **March 2nd, 11:59 p.m.**, not the date on the sticker!
- in testing there was an **occasional** error in submitting, which is fixed by reloading the web page and continuing

fire alarm incident

- about 180 students had their test interrupted by a fire alarm
- our marking scheme has no provision for make-up tests; all likely dates overlap things such as assignment due dates or other course events
- for individuals who miss a test for valid reason, we re-evaluate the mark based on the second test and final
- consulting our department's undergraduate chair, we use a formula we believe neither gives an advantage nor a disadvantage to the affected students (see next slide)

replace test #1 grade

a_1: class average on test #1

a_2: class average on test #2

a_e: class average on on final exam

g_2: student's grade on test #2

g_e: student's grade on final exam

$$\text{test \#1 score:} \quad g_1 = \frac{g_2/a_2 + g_e/a_e}{2} \times a_1$$

rationale: student standing the same compared to the average on test #1 as compared to the average on test #2 and the final

what about those who didn't have a fire alarm?

although we think the formula for those who missed test #1 gives them neither an advantage nor a disadvantage, we will offer the remaining students the maximum of either their current grade on test #1 or the grade calculated using the formula on the previous slide

if a majority of students vote for this change, students who perform better relative to their peers on test #2 and the final may improve their test#1 grade

the vote will be in class, on March 2nd

summing lists

```
L1 = [1, 9, 8, 15]
sum(L1) = ???
L2 = [[1, 5], [9, 8], [1, 2, 3, 4]]
sum([sum(row) for row in L2]) = ??
L3 = [[1, 5], 9, [8, [1, 2], 3, 4]]
```

How can we sum L3?

re-use built-in... recursion!

- a function sum_list that adds all the numbers in a nested list shouldn't ignore built-in sum
- ...except sum wouldn't work properly on the nested lists, so make a list-comprehension of their sum_lists
- but wait, some of the list elements are numbers, not lists!

write a definition of sum_list — don't look at next slide yet!

hey! don't peek!

```
def sum_list(L):
    ''' (list or int) -> int

    Return L if it's an int, or sum of the numbers in possibly nested l

    >>> sum_list(17)
    17
    >>> sum_list([1, 2, 3])
    6
    >>> sum_list([1, [2, 3, [4]], 5])
    15
    '''
    # reuse: isinstance, sum, sum_list !
    if isinstance(L, list):
        return sum([sum_list(x) for x in L])
    else: # L is an int
        return L
```

tracing recursion

To understand recursion, trace from simple to complex:

- trace `sum_list(17)`
- trace `sum_list([1, 2, 3])`. Remember how the built-in sum works...
- trace `sum_list([1, [2, 3], 4, [5, 6]])`. Immediately replace calls you've already traced (or traced something equivalent) by their value
- trace `sum_list([1, [2, [3 ,4], 5], 6 [7, 8]])`. Immediately replace calls you've already traced by their value.

depth of a list

Define the depth of L as 1 plus the maximum depth of L's elements if L is a list, otherwise 0.

- the definition is almost exactly the Python code you write!
- start by writing return and pythonese for the definition:

```
if instance(L, list):
    return 1 + max([depth(x) for x in L])
else: # L is not a list
    return 0
# find the bug! (then fix it...)
```

- deal with the special case of a non-list

trace to understand recursion

Trace in increasing complexity; at each step fill in values for recursive calls that have (basically) **already been traced**

- Trace `depth([])`
- Trace `depth(17)`
- Trace `depth([3, 17, 1])`
- Trace `depth([5, [3, 17, 1], [2, 4], 6])`
- Trace `depth([14, 7, [5, [3, 17, 1], [2, 4], 6], 9])`

maximum number in nested list

Use the built-in `max` much like `sum`

- how would you find the max of non-nested list?

  `max(...)`

- how would you build that list using a comprehension?

  `max([...])`

- what should you do with list items that were themselves lists?

  `max([rec_max(x) ...])`

- get some intuition by tracing through flat lists, lists nested one deep, then two deep...

code for rec_max

```
if isinstance(L, list):
    return max([rec_max(x) for x in L])
else:
    return L
```

trace the recursion

trace from simple to complex; fill in already-solved recursive calls

- trace `rec_max([3, 5, 1, 3, 4, 7])`
- trace `rec_max([4, 2, [3, 5, 1, 3, 4, 7], 8])`
- trace `rec_max([6, [4, 2, [3, 5, 1, 3, 4, 7], 8], 5])`

get some turtles to draw

Spawn some turtles, point them in different directions, get them to draw a little and then spawn again...

Try out tree_burst.py

Notice that **tree_burst** returns **NoneType**: we use it for its side-effect (drawing on a canvas) rather than returning some value.

nested_contains

Return whether a list, or any of its sublists, contain some non-list value.

- should return True if any element is equivalent to value
- should return True if any element is a list ultimately containing value
- Python **any** and functional **if** are useful

```
<expression 1> if <condition> else <expression 2>
```

If the condition is true, evaluates to the first expression, otherwise evaluates to the second expression.

base case, general case

You will have noticed that a recursive function has a conditional structure that specifies how to combine recursive subcalls **(general case)**, and when/how to stop **(the base case, or cases)**.

What happens if you leave out the base case?