CSC148 *Intro. to Computer Science*

Lecture 8: *Binary Trees, BST*

Amir H. Chinaei, Winter 2016

Office Hours: W 16:00–17:45 BA4222

ahchinaei@cs.toronto.edu
http://www.cs.toronto.edu/~ahchinaei/

Course webpage:
http://www.cdf.toronto.edu/~csc148h/winter

Last week

- Tracing recursive programs
- Recursive functions, definitions, and structures
  - Trees
  - Trees terminology ()
- Today
  - Binary trees (arity=2)
  - Binary tree traversals
  - Binary search trees
  - Set

Binary Trees

- Change our generic **Tree** design so that we have two named children, **left** and **right**, and can represent an empty tree with **None**

```
class BinaryTree:
    """
    A Binary Tree, i.e. arity 2.
    """
    def __init__(self, data, left=None, right=None):
        """
        Create BinaryTree self with data & children left & right.

        @param BinaryTree self: this binary tree
        @param object data: data of this node
        @param BinaryTree|None left: left child
        @param BinaryTree|None right: right child
        @rtype: None
        """
        self.data, self.left, self.right = data, left, right
```


Special methods (eq)

```
def __eq__(self, other):
    """
    Return whether BinaryTree self is equivalent to other.

    @param BinaryTree self: this binary tree
    @param Any other: object to check equivalence to self
    @rtype: bool

    >>> BinaryTree(7).__eq__("seven")
    False
    >>> b1 = BinaryTree(7, BinaryTree(5))
    >>> b1.__eq__(BinaryTree(7, BinaryTree(5), None))
    True
    """

    return (type(self) == type(other) and
            self.data == other.data and
            (self.left, self.right) == (other.left, other.right))
```


Special methods (str)

```
def __str__(self, indent=""):
    """
    Return a user-friendly string representing BinaryTree (self)
    inorder.  Indent by indent.
    >>> b = BinaryTree(1, BinaryTree(2, BinaryTree(3)), BinaryTree(4))
    >>> print(b)
        4
    1
        2
            3
    <BLANKLINE>
    """

    right_tree = (self.right.__str__(indent + "    ") if self.right
      else "")

    left_tree = self.left.__str__(indent + "    ") if self.left
      else ""

    return (right_tree + "{}{}\n".format(indent, str(self.data)) +
            left_tree)
```


Special methods (repr)

```
def __repr__(self):
    """
    Represent BinaryTree (self) as a string that can be evaluated to
    produce an equivalent BinaryTree.

    @param BinaryTree self: this binary tree
    @rtype: str

    >>> BinaryTree(1, BinaryTree(2), BinaryTree(3))
    BinaryTree(1, BinaryTree(2, None, None), BinaryTree(3, None, None))
    """

    return "BinaryTree({}, {}, {})".format(repr(self.data),
                                           repr(self.left),
                                           repr(self.right))
```

contains

- you've implemented contains on linked lists, nested Python lists, general Trees before; implement this function, then modify it to become a method

```
def __contains__(self, value):
    """
    Return whether tree rooted at node contains value.

    @param BinaryTree self: binary tree to search for value
    @param object value: value to search for
    @rtype: bool

    >>> BinaryTree(5, BinaryTree(7), BinaryTree(9)).__contains__(7)
    True
    """

    return (self.data == value or
            (self.left and value in self.left) or
            (self.right and value in self.right))
```


arithmetic expression trees

- Binary arithmetic expressions can b e represented as binary trees:

evaluating a binary expression tree

- There are no empty expressions
  - if it's a leaf, just return the value
  - otherwise...
    - evaluate the left tree
    - evaluate the right tree
    - combine left and right with the binary operator
- Python built-in eval might be handy

  >>> eval("2+3")

  5

evaluating a binary expression tree

```
def evaluate(b):
    """
    Evaluate the expression rooted at b.  If b is a leaf,
    return its float data.  Otherwise, evaluate b.left and
    b.right and combine them with b.data.

    Assume:  -- b is a non-empty binary tree
             -- interior nodes contain data in {"+", "-", "*", "/"}
             -- interior nodes always have two children
             -- leaves contain float data

     @param BinaryTree b: binary tree representing arithmetic expression
     @rtype: float

    >>> b = BinaryTree(3.0)
    >>> evaluate(b)
    3.0
    >>> b = BinaryTree("*", BinaryTree(3.0), BinaryTree(4.0))
    >>> evaluate(b)
    12.0
    """

    if b.left is None and b.right is None:
        return b.data
    else:
        return eval(str(evaluate(b.left)) +
                    str(b.data) +
                    str(evaluate(b.right)))
```


inorder

- A recursive definition:
  - visit the left subtree inorder
  - visit this node itself
  - visit the right subtree inorder
- The code is almost identical to the definition.

inorder

```
def inorder_visit(root, act):
    """
    Visit each node of binary tree rooted at root in order and act.
    @param BinaryTree root: binary tree to visit
    @param (BinaryTree)->object act: function to execute on visit
    @rtype: None
    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> def f(node): print(node.data)
    >>> inorder_visit(b, f)
    2
    4
    6
    8
    12
    """

    if root is not None:
        inorder_visit(root.left, act)
        act(root)
        inorder_visit(root.right, act)
```

preorder

- A recursive definition:
  - visit this node itself
  - visit the left subtree preorder
  - visit the right subtree preorder
- The code is almost identical to the definition.

preorder

```
def preorder_visit(root, act):
    """
    Visit each node of binary tree rooted at root in preorder and act.
    @param BinaryTree root: binary tree to visit
    @param (BinaryTree)->object act: function to execute on visit
    @rtype: None
    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> def f(node): print(node.data)
    >>> preorder_visit(b, f)
    8
    4
    2
    6
    12
    """
    if root is not None:
        act(root)
        preorder_visit(root.left, act)
        preorder_visit(root.right, act)
```


postorder

- A recursive definition:
  - visit the left subtree postorder
  - visit the right subtree postorder
  - visit this node itself
- The code is almost identical to the definition.

postorder

```
def postorder_visit(root, act):
    """
    Visit each node of binary tree rooted at root in postorder and act.
    @param BinaryTree root: binary tree to visit
    @param (BinaryTree)->object act: function to execute on visit
    @rtype: None
    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> def f(node): print(node.data)
    >>> preorder_visit(b, f)
    2
    6
    4
    12
    8
    """
    if root is not None:
        postorder_visit(root.left, act)
        postorder_visit(root.right, act)
        act(root)
```


level order

- visit this node
- visit this node's children
- visit this node's grandchildren
- visit this node's great grandchildren
- ...
- Let's have a helper function
  `visit_level (tree, level, act)`

visit_level

```
def visit_level(t, n, act):
    """
    Visit each node of BinaryTree t at level n and act on it.  Return
    the number of nodes visited visited.
    @param BinaryTree|None t: binary tree to visit
    @param int n: level to visit
    @param (BinaryTree)->Any act: function to execute on nodes at level n
    @rtype: int
    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> def f(node): print(node.data)
    >>> visit_level(b, 2, f)
    2
    6
    2
    """
    if t is None: return 0
    elif n == 0:
        act(t)
        return 1
    elif n > 0: return (visit_level(t.left, n-1, act)
                        + visit_level(t.right, n-1, act))
    else: return 0
```

```python
def levelorder(t, act):
    """
    Visit BinaryTree t in level order and act on each node.
    @param BinaryTree|None t: binary tree to visit
    @param (BinaryTree)->Any act: function to use during visit
    @rtype: None
    >>> b = BinaryTree(8)
    >>> b = insert(b, 4)
    >>> b = insert(b, 2)
    >>> b = insert(b, 6)
    >>> b = insert(b, 12)
    >>> def f(node): print(node.data)
    >>> levelorder_visit(b, f)
    8
    4
    12
    2
    6
    """
    # this approach uses iterative deepening

    visited, n = visit_level(t, 0, act), 0
    while visited > 0:
        n += 1
        visited = visit_level(t, n, act)
```


Binary Search Trees

- Add ordering conditions to a binary tree:
  - data are comparable
  - data in left subtree are less than node.data
  - data in right subtree are more than node.data

Binary Search Trees

- a BST with 1 one node has height 1
- a BST with 3 nodes may have height 2
- a BST with 7 nodes may have height 3
- a BST with 15 nodes may have height 4
- a BST with n nodes may have height $\lceil \lg n \rceil$

- if the BST is "balanced", then we can check whether an element is present in about $\lg n$ node accesses

bst_contains

```python
def bst_contains(node, value):
    """
    Return whether tree rooted at node contains value.

    Assume node is the root of a Binary Search Tree

    @param BinaryTree|None node: node of a Binary Search Tree
    @param object value: value to search for
    @rtype: bool

    >>> bst_contains(None, 5)
    False
    >>> bst_contains(BinaryTree(7, BinaryTree(5), BinaryTree(9)), 5)
    True
    """

    if node is None:
        return False
    elif value < node.data:
        return bst_contains(node.left, value)
    elif value > node.data:
        return bst_contains(node.right, value)
    else:
        return True
```


bst_insert

```python
def insert(node, data):
    """
    Insert data in BST rooted at node if necessary, and return new root.

    Assume node is the root of a Binary Search Tree.

    @param BinaryTree node: root of a binary search tree.
    @param object data: data to insert into BST, if necessary.

    >>> b = BinaryTree(5)
    >>> b1 = insert(b, 3)
    >>> print(b1)
    5
        3
    <BLANKLINE>
    """

    return_node = node
    if not node:
        return_node = BinaryTree(data)
    elif data < node.data:
        node.left = insert(node.left, data)
    elif data > node.data:
        node.right = insert(node.right, data)
    else:  # nothing to do
        pass
    return return_node
```


A2 uses sets

a useful data structure

```python
>>> {1, 2, 3} == {2, 3, 1}
True
>>> {1, 2, 3} - {3, 4, 5}
{1, 2}
>>> 2 in {1, 2, 3}
True
>>> {1, 2, 3}.union({3, 4, 5})
{1, 2, 3, 4, 5}
```