CSC148 *Intro. to Computer Science*

Lecture 12: *Efficiency of Recursive Algorithms, big O, Hash Table, Final Exam.*

Amir H. Chinaei, Winter 2016

Office Hours: W 16:00–17:45 BA4222

ahchinaei@cs.toronto.edu

http://www.cs.toronto.edu/~ahchinaei/

Course webpage:

http://www.cdf.toronto.edu/~csc148h/winter

Review

- Efficiency of ***iterative*** algorithms
  - In CSC148, we mainly focus on time efficiency
    - i.e. **time complexity**
  - We calculate/estimate a function denoting the number of operations (e.g. comparisons), and we focus on the ***dominant term***:
    - discard all irrelevant coefficients as well as all non-dominant terms
  - We focus on the ***loops***
    - The way the ***loop invariant*** is changed
    - If the loops are ***nested*** or ***sequential***
  - We also watch the ***function calls***

Efficiency of recursive algorithms?

Example 1: BST Contains

A divide and conquer problem:

```
def bst_contains(node, value):
    if node is None:
        return False
    elif value < node.data:
        return bst_contains(node.left, value)
    elif value > node.data:
        return bst_contains(node.right, value)
    else:
        return True
```

- Denote T(n) as the number of operations for a tree with *n* nodes
- Assume we always have the best tree:
  - i.e the tree is (almost) balanced
- **$T(n)=T(n/2) + \varepsilon$**
- We will see the big O notation of this, shortly.

Example 2: Quick Sort

Another divide and conquer problem:

```
Qsort (A, i, j)
if (i < j)
    p := partition(A)
    Qsort (A, i, p-1)
    Qsort (A, p+1, j)
end
```

- Denote T(n) as the number of operations in Qsort for a list with *n* items
- Partition requires to traverse the whole list, i.e. ***n*** iterations
- Assume we have the best partition function: i.e. *p* is roughly at the middle of the list
- **$T(n)=n+ 2T(n/2) + \varepsilon$**
- We will see the big O notation of this, shortly.

Example 3: Merge Sort

Another, divide and conquer problem:

```
Msort (A, i, j)
if (i < j)
    S1 := Msort(A, i , (i+j)/2)
    S2 := Msort(A, (i+j)/2, j)
    Merge(S1,S2, i, j)
end
```

- Denote T(n) as the number of operations in Msort for a list with *n* items
- Merge is to merge two sorted lists in one: the result will have n items. hence, Merge requires ***n*** operations
- The list will be always halved
- **$T(n)=2T(n/2) + n + \varepsilon$**
- We will see the big O notation of this, shortly.

big O of recurrence relations

- It's covered in CSC236
  - For instance, via the Master Theorem
  - If interested, read the following:
  - Let T be an increasing function that satisfies the recurrence relation

    $T(n) = a\ T(n/b) + cn^d$

    whenever $n = b^k$, where k is a positive integer greater than 1, and c and d are real numbers with c positive and d nonnegative. Then

$$T(n) \text{ is } \begin{cases} O(n^d) & \text{if} \quad a < b^d, \\ O(n^d \log n) & \text{if} \quad a = b^d, \\ O(n^{\log_b a}) & \text{if} \quad a > b^d. \end{cases}$$

big O of recurrence relations

- For now, we are going to accept the following common ones:

Recurrence Relation	***Time Complexity***	***Example Algorithms***
$T(n)=T(n/2)+O(1)$	$T(n) \in O(\log n)$	bst_contains, Binary Search
$T(n) = T(n-1) + O(1)$	$T(n) \in O(n)$	Factorial
$T(n) = 2T(n/2) + O(n)$	$T(n) \in O(n \log n)$	Qsort, Msort
$T(n) = T(n-1)+T(n-2)+O(1)$	$T(n) \in 2^n$	Recursive Fibunacci

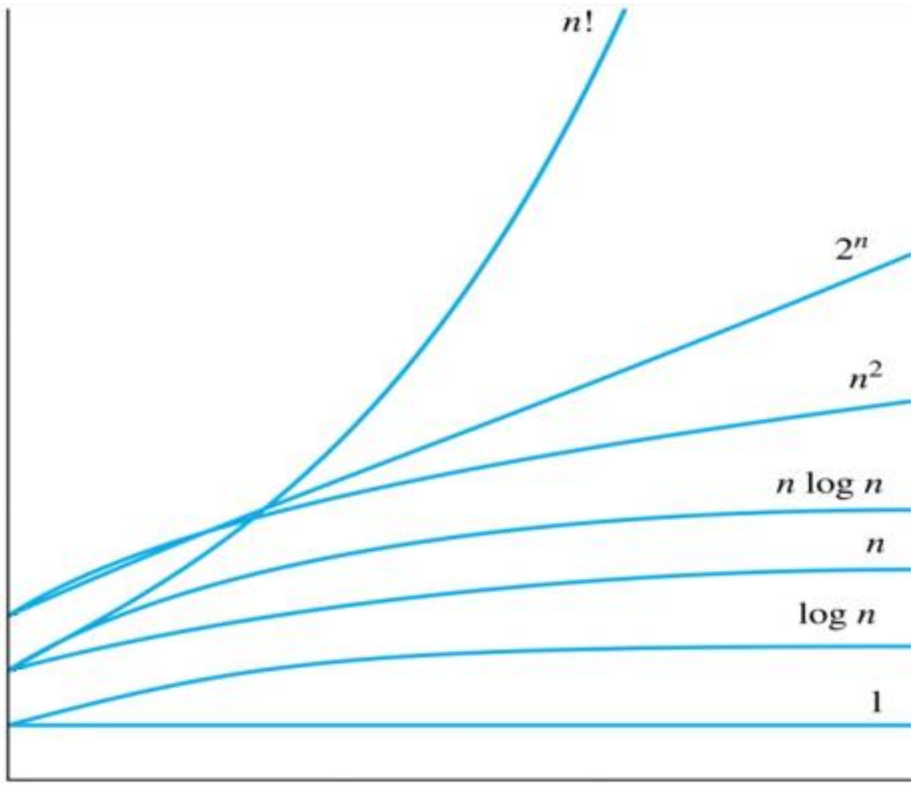
More insight to big O

- When we say an algorithm (or a function) $f(n)$ is in $O(g(n))$, we mean $f(n)$ is bounded (from up) by $g(n)$. In other words, $g(n)$ is an upper bound for $f(n)$

- This means, there are positive constants c and n_0 such that
  $f(n) \le c\, g(n)$ for all $n > n_0$

- Intuitively, this means that $f(n)$ grows slower than some fixed multiple of $g(n)$ as n grows without bound.

Recall



So, we can say:

...

2^n is an upper bound for n^2

and

n^2 is an upper bound for $n \log n$,

and

$n \log n$ is an upper bound for n,

...

Find c and n_0 for each of these cases

big O

If a function $\in O(n)$, it's also $\in O(n \log n)$ and $\in O(n^2)$

In general,

$O(1) \subseteq \ldots \subseteq O(\log \log n) \subseteq O(\log n) \subseteq O(n \log n) \ldots \subseteq O(n^2) \subseteq \ldots \subseteq$

$\subseteq O(n^2 \log n) \ldots \subseteq O(n^3) \subseteq \ldots \subseteq O(n^4) \ldots \subseteq O(2^n) \ldots \subseteq O(3^n) \ldots \subseteq O(n!)$

However, when are looking for an upper bound, we are required to find the tightest one

$F(n) = 5\, n^2 + 1000$ is in $O(n^2)$

Recall: Python lists and our liked lists

- Python list is a contiguous data structure
  - *Lookup* is fast
  - *Insertion* and *deletion* is slow

- linked list is not a contiguous data structure
  - *Lookup* is slow
  - *Insertion* and *deletion* (when does not require lookup) is fast

	lookup	insert	delete
Lists	O(1)	O(n)	O(n)
Linked Lists	O(n)	O(1)	O(1)

Recall: Balanced BST

- BST can be implemented by linked lists
- Yet, it has a property that makes it more efficient when it comes to lookup

	lookup	insert	delete
Lists	O(1)	O(n)	O(n)
Linked Lists	O(n)	O(1)	O(1)
BST	O(log n)	O(log n)	O(log n)

- Yet, this comes at a price for insertion and deletion

- **Can we do better?**

Can we do better?

- Assume a magical machine:
  - Input: a **key**
  - Output: its **index** value in a list

- Well, this is a mapping machine:
  - A pair of (**key**, **index**)
  - The **key** is the value that we want to lookup or insert or delete, and the **index** is its location in the list

- And, it's called a **hash function**

Hash Function

- A hash function first *converts* a key to an integer value,
- Then, *compresses* that value into an index.

- Just as a simple example:
- The *conversion* can be done by applying some functions to the binary values of the characters of the key

- And the compression can be done by some modular operations.

Example: (insertion)

- A class roster of up to 10 students:
  - We want to enroll “ANA”
  - Hash function:
    - *Conversion* component, for instance, returns 208 which is 65+78+65
    - *Compression* component, for instance, returns 8 which is 208 mod 10
  - So, we insert “ANA” at index 8 of the roster.

  - Similarly, if we want to enroll “ADAM”,
    - we insert it at index 5 of the roster (let’s call it hash table).

Example: (lookup)

- We want to lookup "ANA"
- Hash function:
  - *Conversion* component, for instance, returns 208 which is 65+78+65
  - *Compression* component, for instance, returns 8 which is 208 mod 10
- So, we check index 8 of the roster.

- Similarly, if we want to lookup "ADAM",
  - we check index 5 of the roster (hash table).

Recall: Balanced BST

	lookup	insert	delete
Lists	O(1)	O(n)	O(n)
Linked Lists	O(n)	O(1)	O(1)
BST	O(log n)	O(log *n*)	O(log *n*)
Hash Table	O(1)*	O(1) *	O(1) *

* if there is no collision

Collision

- How collision can happen?

- What can we do when there is a collision?
  - Chaining

  - Probing

  - Double hashing