These exercises are intended to give you some practice applying the Master Theorem\(^1\) to algorithm design.

1. Consider the following sketch of a divide-and-conquer algorithm \( r(s) \) for reversing a string:
   
   (a) \( s \) is a string.
   (b) If \( \text{len}(s) < 2 \), return \( s \)
   (c) Else, partition \( s \) into three roughly equal parts: prefix \( s_1 \), suffix \( s_3 \), and mid-section \( s_2 \), and return \( r(s_3) + r(s_2) + r(s_1) \).
   (d) You may assume that the time complexity of string concatenation of \( s_3 + s_2 + s_1 \) is proportional to \( \text{len}(s_3) + \text{len}(s_2) + \text{len}(s_1) \)

   Use the Master Theorem to find the asymptotic time complexity of function \( r \) in terms of \( \text{len}(s) \). Be sure to show all the components of your analysis, including the values of \( a, b \), and \( d \). How does this compare to the complexity of simply copying the string elements in reverse order, using a loop?

2. Describe a ternary version of MergeSort where the list segment to be sorted is divided into three (roughly) equal sub-lists, rather than two. Use the Master Theorem to find the asymptotic time complexity of your ternary MergeSort in terms of the length of the list segment being sorted, and compare/contrast it with the version we analyzed in class. Be sure to show all the components of your analysis, including the values of \( a, b \), and \( d \).

3. Consider the following sketch of bisection algorithm \( \text{bis}(f, a, b, \gamma, \delta) \) to approximate a root of a function:
   
   (a) \( f : \mathbb{R} \to \mathbb{R} \) is a function, \( a, b \in \mathbb{R} \) with \( f(b) \times f(a) \leq 0 \), \( \gamma, \delta \in \mathbb{R^+} \)
   (b) If \( |b - a| < \gamma \) return \((a + b)/2\).
   (c) If \( |f(a)| < \delta \) return \( a \).
   (d) If \( |f(b)| < \delta \) return \( b \).
   (e) If \( f(a) \times f([a + b]/2) \leq 0 \) return \( \text{bis}(f, a, (a + b)/2, \gamma, \delta) \).
   (f) Otherwise return \( \text{bis}(f, (a + b)/2, b, \gamma, \delta) \)

   Use the Master Theorem to find the asymptotic time complexity of function \( \text{bis} \) in terms of \( |b - a|/\gamma \). Be sure to show all the components of your analysis.

\(^1\)Very abbreviated version on next page...
\[ T(n) = \begin{cases} 
  k & \text{if } n \leq b \\
  a_1 T(\lfloor n/b \rfloor) + a_2 T(\lceil n/b \rceil) + f(n) & \text{if } n > b
\end{cases} \]

\[ T(n) \in \begin{cases} 
  \Theta(n^a) & \text{if } a < b^d \\
  \Theta(n^a \log_b n) & \text{if } a = b^d \\
  \Theta(n^{\log_b a}) & \text{if } a > b^d
\end{cases} \]