Solve question 1, then prove claims 2–4 using Mathematical Induction (aka Simple Induction).

1. Define $P(n)$ as:
   \[ \sum_{i=0}^{\infty} 2^i = 2^{n+1} \]
   (a) Prove that $P(115)$ implies $P(116)$.
   (b) Is $P(n)$ true for every natural number $n$? Explain why, or why not.

2. $\forall n \in \mathbb{N}, 8n - 1$ is a multiple of 7.

3. $\forall n \in \mathbb{N}, \exists m \in \mathbb{N},$ the units digit of $7^n$ is the same as the units digit of $3^m$.

4. $\exists m \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq m \Rightarrow 4^n \geq 5n^4 + 6$