These exercises are intended to give you some practice devising deterministic finite state automata (DFAs).

1. Let $L_1 = \{ x \in \{a,b\}^* \mid \text{the number of } a \text{ in } x \text{ is even}\}$, and let $L_2 = \{ z \in \{a,b\}^* \mid |z| \equiv 0 \mod 3\}$. Build DFAs that accept $L_1$, $L_2$, and use the product procedure to build a DFA that accepts $L_1 \cap L_2$.

2. Use structural induction to prove that the DFAs you propose accept $L_1$ and $L_2$. Without any further induction, prove that your product machine accepts $L_1 \cap L_2$ by constructing a state invariant consisting of conjunctions of the state invariants of the other two machines, and then using your earlier proofs to show that this new state invariant is correct.