UNIVERSITY OF TORONTO

Faculty of Arts and Science

term test #1, Version 2

CSC165H1S

Date: Tuesday October 10, 3:10–4:00pm

Duration: 50 minutes

Instructor(s): Danny Heap

No Aids Allowed

Name:

utorid:

U of T email:

Please read the following guidelines carefully!

- **Please write your name on both the front and back of this exam.**
- This examination has **4** questions. There are a total of **8 pages, DOUBLE-SIDED**.
- Answer questions clearly and completely. Provide justification unless explicitly asked not to.
- All formulas must have negations applied directly to propositional variables or predicates.
- In your proofs, you may always use definitions of predicates from the course. You may *not* use any external facts about rates of growth, divisibility, primes, or greatest common divisor unless you prove them, or they are given to you in the question.

Take a deep breath.
This is your chance to show us
How much you've learned.

Good luck!

1. **[6 marks] Statements in logic.**

(a) **[3 marks]** Write the truth table for the following formula. No rough work is required.

$$\big((p \Rightarrow q) \vee \neg r\big) \Leftrightarrow \neg p$$

(b) **[3 marks]** Consider the pair of statements:

(1) $\exists n \in \mathbb{N},\ P(n) \Leftrightarrow Q(n)$ (2) $\exists n \in \mathbb{N},\ P(n) \wedge Q(n)$

Define the predicates P and Q with domain $\mathbb{N}$ so that one of these statements is true and the other one false. Note that you're only defining the predicates *once*: the two statements must use the same definitions for P and Q. Also, briefly explain which statement is true and which one is false, and why; no formal proofs necessary.

2. **[7 marks] Translating statements.**

A **powerful number** is a positive integer m such that for every prime p that divides m, p^2 also divides m.

Express each of the following statements using predicate logic. No justification is required. Note: please review the instructions on the midterm's front page for our expectations in this question. In particular, you may *not* define any helper predicates or sets.

(a) 24 is not a powerful number.

(b) 81 is the smallest powerful number greater than 72.

3. **[6 marks] Proofs (inequalities).** Consider the following statement: "For every natural number x there is a natural number y such that $15 > xy > 5$."

   (a) **[1 mark]** Translate the above statement into predicate logic. Use the symbol to denote the set of positive real numbers.

   (b) **[1 mark]** Write the negation of this statement, fully-simplified so that all negation symbols are applied directly to predicates. You can simplify $\neg(a > b)$ to $a \leq b$.

   (c) **[4 marks]** Disprove the original statement by proving its negation. In your proof, any chains of calculations must follow a top-down order; don't start with the inequality you're trying to prove!

   Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be looked at if your proof is not completely correct.

Discussion.

Proof.

4. **[5 marks] Proofs (number theory).** Consider the following statement: "If m and n are integers, and 5 divides both m and n, then 5 divides $2m + n$."

   (a) **[1 mark]** Translate the above statement into predicate logic.

   (b) **[4 marks]** Prove the above statement using the definition of divisibility:

   $$x \mid y : \ \exists k \in \mathbb{Z}, \ y = kx$$

   Do not use any external facts about divisibility.

   Write any rough work or intuition in the **Discussion** box, and write your formal proof in the **Proof** box. Your rough work/intuition will only be looked at if your proof is not completely correct.

Discussion.

Proof.

This page is left nearly blank for rough work. If you want work on this page to be marked, please indicate this clearly *at the location of the original question.*

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Name:

Question	Grade	Out of
Q1		6
Q2		7
Q3		6
Q4		5
Total		24