

# CSC165 fall 2019

worst/best/average

later  
263

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BA4270 (behind elevators)

Web page:

<http://www.teach.cs.toronto.edu/~heap/165/F19/>

Using Course notes: algorithm analysis

frequently asked...

≈ fP. 93 - 95.

```
def all_pairs(lst: list) -> None:
```

- $i = 0$

```
    while i < len(lst): — n = len(lst)
```

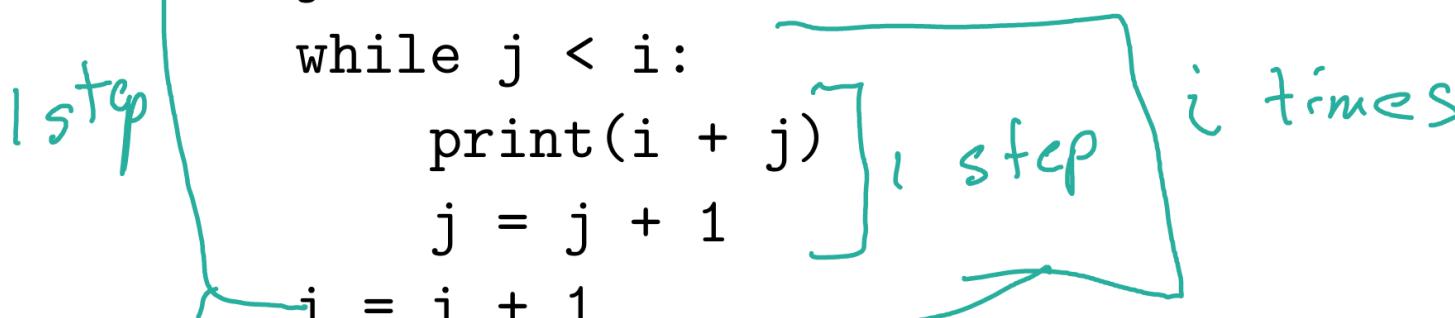
```
        j = 0
```

```
        while j < i:
```

```
            print(i + j)
```

```
            j = j + 1
```

```
        i = i + 1
```



$$RT_{all\_pairs}(n) = \sum_{i=0}^{n-1} (i+1) = \sum_{i'=1}^n i' = \frac{n(n+1)}{2} + n \in \Theta(n^2).$$

Try analysis

$$\left( \sum_{i=0}^{n-1} i \right) + 2^n + 1$$



compare...

$$L(n) \in \Omega(n)$$
$$U(n) \in O(n)$$

```
def is_prime(n):  
    if n < 2:  
        return False  
    else:  
        for d in range(2, n):  
            if n % d == 0:  
                return False  
    return True
```

$$L(n) \downarrow \left\lceil \frac{n}{2} \right\rceil + 1$$

best, worst,  
average steps  
of size  $n$   
is the same

$$\text{has\_even}([1]^* \left\lceil \frac{n}{2} \right\rceil + [2]^* \left\lfloor \frac{n+1}{2} \right\rfloor) = \text{len}(\text{number\_list})$$

```
def has_even(number_list):  
    for number in number_list:  
        if number % 2 == 0:  
            return True  
    return False
```

1 step

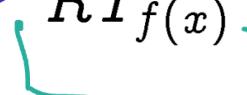
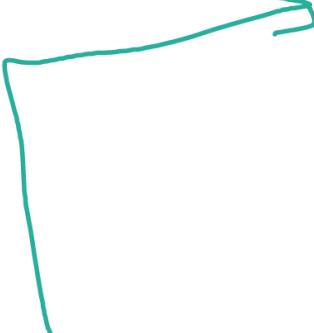
has\_even([2]^\* 10^00)  
has\_even([1]^\* 10^00)

iterations

$$U(n) = n+1$$

## definitions

set (family) of inputs of  
size  $n$ .

- ▶  $\mathcal{I}_{f,n} = \{i \mid i \text{ is an input to } f \wedge |i| = n\}$   
  

- ▶  $RT_{f(x)}$  = number of basic “steps” in executing  $f(x)$
- ▶  $WC_f(n) = \max\{RT_{f(x)} \mid x \in \mathcal{I}_{f,n}\}$

upper bounds, lower bounds...

on worst case

- ▶  $U(n)$  is an upper bound means  
 $\forall n \in \mathbb{N}, \forall x \in \mathcal{I}_{f,n}, RT_{f(x)} \leq U(n)$

?

- ▶  $L(n)$  is a lower bound means  
 $\forall n \in \mathbb{N}, \exists x \in \mathcal{I}_{f,n}, RT_{f(x)} \geq L(n)$

why the asymmetry of  $U$  and  $L$ ?

$RT_f$ :

• • • • • • • • • • • • • •

$L(n)$

$U(n)$