CSC165 fall 2019

worst/best/average

later 263

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Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: algorithm analysis

frequently asked...

```
def all_pairs(lst: list) -> None:
    i = 0
    while i < len(lst):
        j = 0
        while j < i:
            print(i + j)
            j = j + 1
        i = i + 1
```

$$RT_{all\_pairs}(n) = \sum_{i=0}^{n-1}(i+1) = \sum_{i'=1}^{n} i' = \frac{n(n+1)}{2} + n \in \Theta(n^2).$$

compare...

$L(n) \in \Omega(n)$
$U(n) \in O(n)$

best, worst, average steps of size n is the same

```
def is_prime(n):
    if n < 2:
        return False
    else:
        for d in range(2,n):
            if n % d == 0:
                return False
        return True
```

$L(n) = \lceil \frac{n}{2} \rceil + 1$

has_even([1]*$\lceil \frac{n}{2} \rceil$ + [2]*$\lfloor n/2 \rfloor$) n = len(number_list)

```
def has_even(number_list):
    for number in number_list:
        if number % 2 == 0:
            return True
    return False
```

1 step

≤ n iterations

+1

$U(n) = n+1$

has_even([2]*1000)
has_even([1]*1000)

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definitions

set (family) of inputs of size n.

- $\mathcal{I}_{f,n} = \{i \mid i \text{ is an input to } f \wedge |i| = n\}$

size

- $RT_{f(x)}$ = number of basic "steps" in executing $f(x)$

- $WC_f(n) = \max\{RT_{f(x)} \mid x \in \mathcal{I}_{f,n}\}$

upper bounds, lower bounds...

on worst case

WC$_f$(n)

somewhere between U(n) and L(n)

- $U(n)$ is an upper bound means
  $\forall n \in \mathbb{N}, \forall x \in \mathcal{I}_{f,n}, RT_{f(x)} \leq U(n)$

?

- $L(n)$ is a lower bound means
  $\forall n \in \mathbb{N}, \exists x \in \mathcal{I}_{f,n}, RT_{f(x)} \geq L(n)$

why the asymmetry of U and L?