CSC165 fall 2019

end induction...
...begin algorithm analysis

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Using Course notes: more Induction

Outline

notes

every set with n elements has 2^n subsets

order of introductions... and intuition

taking binary representation apart

suppose n is a natural number with binary representation:

$$n = \Sigma_{i=0}^{i=p} b_i 2^i = b_p 2^p + b_{p-1} 2^{p-1} + \cdots + 2^1 b_1 + 2^0 b_0$$

figure out b_0 based on whether n is odd or even... suggesting

```
def natural_to_binary(n: int) -> str:
    # convert n to equivalent binary string
    bs = str(n % 2)
    n = n // 2
    while n > 0:
        bs = str(n % 2) + bs
        n = n // 2
    return bs
```

time resource

How much time does this take?

```
def f(list_):
    for i in list_:
        print(i)
```

assumptions, assumptions...

- “steps”
- ignore constant factors
- ignore “noise” for small input

We care about growth rate of time consumption

formalizing assumptions

- f absolutely dominates g
- f dominates g up to a constant factor
- f eventually dominates g up to a constant factor

What should domain and range of f, g be?

big-Oh, big-Omega, big-Theta

... and you're started on the Greek alphabet...

$$f \in \mathcal{O}(g) : \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow f(n) \leq cg(n)$$

try to show that $\forall a, b \in \mathbb{R}^+, an + b \in \mathcal{O}(n^2)$

$$\forall a, b \in \mathbb{R}^+, an + b \in \mathcal{O}(n^2)$$

big-Oh hierarchy when $0 < a < b$

recall $\mathcal{O}(f) = \{g \mid g : \mathbb{N} \to \mathbb{R}^+ \wedge \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq cf(n)\}$

$\log_a n$ versus $\log_b n$ (logarithmic)

n^a versus n^b (polynomial)

a^n versus b^n (exponential)

$\log_a n$ versus n^a

n^a versus b^n

explore!

properties

- reflexivity
- transitivity of big-Oh
- **not** symmetry (anti-symmetry...)

products and sums

- af
- $f \cdot g$
- $f + g$