PS1: Soon... a couple of TAs still grading

PS2: Suggestions for FAQ; hints welcome

CSC165 fall 2019

Mathematical expression: induction

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Web page:

http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: Induction

more domino logistics....

Sometimes the dominoes don't start falling from 0

$7^0 \equiv 1 \pmod 6$ $\quad 7^1 \equiv 1 \pmod 6$ $\quad 7^2 \equiv 1 \pmod 6$ $\quad 7^3 \equiv 1 \pmod 6$ $\quad 7^4 \equiv 1 \pmod 6$ $\quad 7^5 \equiv 1 \pmod 6$ $\quad \ldots$ "etc."

fall ≥ 2

$7^0 \equiv 1 \pmod 6$ $\quad 7^1 \equiv 1 \pmod 6$ $\quad 7^2 \equiv 1 \pmod 6$ $\quad 7^3 \equiv 1 \pmod 6$ $\quad 7^4 \equiv 1 \pmod 6$ $\quad 7^5 \equiv 1 \pmod 6$ $\quad \ldots$ "etc."

start at 2

$2n + 1 < 2^n$ $\qquad$ $P(n): n \geq 3 \Rightarrow 2n+1 < 2^n$

<u>base case $P(3)$</u>: $2 \times 3 + 1 = 6+1 = 7 < 8 = 2^3$,

so $P(3)$ is verified.

<u>inductive step</u> Let $n \in \mathbb{N}$. Assume $n \geq 3$ and assume $P(n)$, so $2n+1 < 2^n$. WTS $P(n+1)$,

i.e. $2(n+1)+1 < 2^{n+1}$.

$$
\begin{aligned}
2(n+1)+1 &= 2n+2+1 = \underline{2n+1} + 2 \\
&< 2^n + 2 && \# \text{ by IH} \\
&< 2^n + 2^n && \# \; n \geq 3,\ 2^n \geq 8 > 2 \\
&= 2^{n+1} \quad \blacksquare
\end{aligned}
$$

$\forall n \in \mathbb{N}, 3^n \geq n^3$

$P(n)$: $n \geq 3 \Rightarrow 3^n \geq n^3$

inductive step: Let $n \in \mathbb{N}$. assume $P(n)$, $\wedge$ $n \geq 3$

So $3^n \geq n^3$. WTS $P(n+1)$, that is $3^{n+1} \geq (n+1)^3$.

$$
\begin{aligned}
3^{n+1} &= 3 \cdot 3^n = 3^n + 3^n + 3^n \\
&\geq n^3 + n^3 + n^3 && \text{\# by IH} \\
&\geq n^3 + 3n^2 + 9n && \text{\# } n \geq 3 \\
&= n^3 + 3n^2 + 3n + 6n && \text{\# } n \geq 1/6 \\
&\geq n^3 + 3n^2 + 3n + 1 \\
& \\
n^3 + 3n^2 + 3n + 1 & && \text{\# binomial} \\
&= && \text{\# thm} \\
&= (n+1)^3 \quad \blacksquare
\end{aligned}
$$

base case, $P(3)$ $3^3 \geq 3^3$, so $P(3)$ is verified.

$\forall x, y \in \mathbb{N}, \forall n \in \mathbb{N}, x - y \mid x^n - y^n$

order of introductions...

Ex 3.14 - either fix (x,y) & prove $\forall n$, or fix n, prove $\forall (x,y)$.

Let $x, y \in \mathbb{N}$. $P(n): x-y \mid x^n - y^n$

Proof, by induction, $\forall n \in \mathbb{N}, P(n)$.

base case, $P(0)$ $x^0 - y^0 = 1 - 1 = 0 = 0 \cdot (x-y)$

so $P(0)$ holds.

inductive step: Let $n \in \mathbb{N}$. assume $P(n)$, so $x-y \mid x^n - y^n$. $\exists k \in \mathbb{Z}, (x-y)k = x^n - y^n$. Let $k_2 = \underline{xk + y^n}$. WTS $k_2(x-y) = x^{n+1} - y^{n+1}$.

$$
\begin{aligned}
x^{n+1} - y^{n+1} &= x(x^n - y^n) + y^n(x-y) \\
&= xk(x-y) + y^n(x-y) \\
&= k_2(x-y) \quad \blacksquare
\end{aligned}
$$

every set with n elements has 2^n subsets

more order of introductions...