Prep 4 quiz

ⓘ This is a preview of the published version of the quiz

Started: Sep 19 at 12:17pm

Quiz Instructions

Readings

Please read the following parts of the **Course Notes.** **(http://www.teach.cs.toronto.edu/~heap/165/F19/Notes/csc165.pdf)**

- Chapter 2, pp. 38–60

General instructions

You can review the general instructions for all prep quizzes at this page. Remember that you can submit multiple times! You might consider printing this quiz out so that you can work on paper first.

Question 1 **1 pts**

Here are several different statements in predicate logic:

1. $\forall n, k \in \mathbb{N},\ Q(n, k)$
2. $\forall n \in \mathbb{N},\ \exists k \in \mathbb{N},\ Q(n, k)$
3. $\forall n \in \mathbb{N},\ P(n)$
4. $\forall n, p \in \mathbb{N},\ Prime(p) \land p \mid n \Rightarrow Q(n, p)$
5. $\forall a, b \in \mathbb{N},\ \forall n \in \mathbb{N},\ a \mid n \land b \mid n \land P(n)$
6. $\forall n \in \mathbb{N},\ 5 \mid n \land P(n)$
7. $\forall n \in \mathbb{N},\ 5 \mid n \Rightarrow P(n)$
8. $\exists a, b \in \mathbb{N},\ \forall n \in \mathbb{N},\ a \mid n \land b \mid n \Rightarrow P(n)$

For each proof header below, select the statement from the list above that is being proved.

Let n be an arbitrary natural number. Assume that 5 divides n.	[Choose]
Let n and p be arbitrary natural numbers. Assume p is prime and that n is divisible by p.	[Choose]
Let n be an arbitrary natural number, and let k = 3n + 1.	[Choose]
Let a = 7 and b = 9. Let n be an arbitrary natural number, and assume that a divides n and that b divides n.	[Choose]

Question 2

1 pts

Recall the definition of **prime** from lecture, which can be expressed by the following predicate over $\mathbb{N}$:

$$Prime(p): \quad p > 1 \land \left(\forall d \in \mathbb{N},\ d \mid p \Rightarrow d = 1 \lor d = p\right)$$

Suppose we want to *prove* the following statement: "for every positive integer n, the value $n^2 + n + 1$ is prime."

What would be the appropriate proof structure for this proof? (Hint: translate the statement into predicate logic first!)

- Let $n \in \mathbb{Z}^+$. We want to prove that $n^2 + n + 1$ is prime, i.e., that $\forall d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \vee d = n^2 + n + 1$.
  Let $d \in \mathbb{N}$ and assume that $d \mid (n^2 + n + 1)$.
  We will prove that $d = 1$ or that $d = n^2 + n + 1$.

  [Proof body...]

- Let $n \in \mathbb{Z}^+$. Assume $n > 1$ and $d \mid n^2 + n + 1$.
  We will prove that $d = 1$ or that $d = n^2 + n + 1$.

  [Proof body...]

- Let $n \in \mathbb{Z}^+$. We want to prove that $n^2 + n + 1$ is prime, i.e., that $n^2 + n + 1 > 1$ and $\forall d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \vee d = n^2 + n + 1$.

  Part 1: we prove that $n^2 + n + 1 > 1$.

  [Proof body for Part 1...]

  Part 2: we prove that $\forall d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \vee d = n^2 + n + 1$.
  Let $d \in \mathbb{N}$, and assume that $d \mid (n^2 + n + 1)$. We will prove that $d = 1$ or that $d = n^2 + n + 1$.

  [Proof body for Part 2...]

- Let $n \in \mathbb{Z}^+$. We want to prove that $n^2 + n + 1$ is prime, i.e., that $n^2 + n + 1 > 1$ and $\forall d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \vee d = n^2 + n + 1$.

  Part 1: we prove that $n^2 + n + 1 > 1$.

  [Proof body for Part 1...]

  Part 2: we prove that $\forall d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \vee d = n^2 + n + 1$.
  Let $d \in \mathbb{N}$. We divide this proof into cases.

  Case 1: assume that $d = 1$.

  [Proof body in Case 1...]

  Case 2: assume that $d = n^2 + n + 1$.

  [Proof body in Case 2...]

Question 3 — 1 pts

Using the definition of *prime* from the previous question, select the correct *negation* of the $Prime$ predicate below.

Hint: before even looking at the responses, use the negation rules on the definition of the $Prime$ predicate. If you do this correctly, you should see your result in the list below.

- $p \leq 1 \lor (\exists d \in \mathbb{N},\ d \mid p \land d \neq 1 \land d \neq p)$
- $p > 1 \land (\forall d \in \mathbb{N},\ d \mid p \Rightarrow d = 1 \lor d = p)$
- $p \leq 1 \lor (\exists d \in \mathbb{N},\ d \mid p \Rightarrow d = 1 \land d = p)$
- $p > 1 \land (\exists d \in \mathbb{N},\ d \mid p \land d \neq 1 \land d \neq p)$

Question 4

1 pts

Suppose we want to *prove* the following statement: "for every positive integer n, the value $n^2 + n + 1$ is not prime."

What would be the appropriate proof structure for this proof? (Hint: translate the statement into predicate logic first!)

( ) Let $n \in \mathbb{Z}^+$. We will prove that $n^2 + n + 1$ is not prime. Since $n^2 + n + 1 > 1$, we'll need to prove the second part of the "or", i.e., that
$\exists d \in \mathbb{N},\ d \mid (n^2 + n + 1) \Rightarrow d = 1 \lor d = n^2 + n + 1$.
Let $d =$ _____. We want to prove that
$d \mid (n^2 + n + 1) \Rightarrow d = 1 \lor d = n^2 + n + 1$.

Then since $d \nmid (n^2 + n + 1)$, the implication is vacuously true.

( ) Let $n = 10$. We will prove that $n^2 + n + 1$ is not prime. Since $n^2 + n + 1 > 1$, we'll need to prove the second part of the "or", i.e., that
$\exists d \in \mathbb{N},\ d \mid (n^2 + n + 1) \land d \neq 1 \land d \neq n^2 + n + 1$.
Let $d = 3$.

Part 1: we prove that $d \mid (n^2 + n + 1)$.

[Proof body for Part 1...]

Part 2: we prove that $d \neq 1$.

[Proof body for Part 2...]

Part 3: we prove that $d \neq n^2 + n + 1$.

[Proof body for Part 3...]

( ) Let $n \in \mathbb{Z}^+$. We will prove that $n^2 + n + 1$ is not prime. Since $n^2 + n + 1 > 1$, we'll need to prove the second part of the "or", i.e., that
$\exists d \in \mathbb{N},\ d \mid (n^2 + n + 1) \land d \neq 1 \land d \neq n^2 + n + 1$.
Let $d =$ _____.

Part 1: we prove that $d \mid (n^2 + n + 1)$.

[Proof body for Part 1...]

Part 2: we prove that $d \neq 1$.

[Proof body for Part 2...]

Part 3: we prove that $d \neq n^2 + n + 1$.

[Proof body for Part 3...]

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