ps1 - due 1 week + — office hour WMW

+1 - 2 weeks

CSC165 fall 2019

Mathematical expression: proofs

Danny Heap
csc165-2019-09@cs.toronto.edu
BA4270 (behind elevators)
Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/
416-978-5899

Using Course notes: Proof

prove $n^2 - 5n > 7$ for most $n \in \mathbb{N}$

$\forall n \in \mathbb{N},\ n \geq 7 \Rightarrow n^2 - 5n > 7$

discuss

Since $n \geq 7$, $n^2 \geq 7n$ & so

$n^2 - 5n \geq 7n - 5n = 2n \geq 14 > 7.$

Proof

Let $n \in \mathbb{N}$ (same as "let n be an arbitrary fixed n.n.").

Assume $n \geq 7$. WTS $n^2 - 5n > 7$.

** $n \geq 7$ # assumed

* $n - 5 \geq 2$ # since $n \geq 7$

· $n(n-5) \geq 2n$ # mult * by $n > 0$

$2n \geq 14 > 7$ ■ # mult ** by 2

prove $n \mid n+3 \Rightarrow n \mid 3$ for $n \in \mathbb{N}$

$\forall n \in \mathbb{N}, [\exists k_1 \in \mathbb{Z}, n+3 = k_1 n] \Rightarrow \exists k_2 \in \mathbb{Z}, 3 = k_2 n$

discuss Since $n+3 = k_1 n$, then $3 = k_1 n - n$
$= (k_1 - 1) n$

So $k_2 = k_1 - 1$ seems to work!

Proof Let $n \in \mathbb{N}$. assume $\exists k_1 \in \mathbb{Z}$, such that $n+3 = k_1 n$. Let $k_2 = k_1 - 1$. WTS $3 = k_2 n$

$k_2 n = (k_1 - 1) n$

$= k_1 n - n$

$= n+3-n$ # assumption

$= 3$ ∎

generalize $n \mid n + \cancel{3} \Rightarrow n \mid \cancel{3}$ for $n \in \mathbb{N}$

$\forall n, d \in \mathbb{N}$, $n \mid n+d \overset{d}{\Rightarrow} n \mid d \overset{d}{}$.

discussion: if I replace all 3s in previous proof by ds, still follows.

Proof: Modify (mutatis mutandis) previous proof.

prove $m, n \equiv 1 \bmod 3 \Rightarrow mn \equiv 1 \bmod 3$

$m \equiv 1 \bmod 3$ say "m congruent to 1 mod 3"

mean $3 \mid (m-1)$