ps0 – sample solution posted
ps1 – posted, due in < 2 weeks

CSC165 fall 2019

Mathematical expression: proofs

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Using Course notes: Proof

$a \mid b$ and $Prime(p)$
statements packed and unpacked

divides(a, b): $\exists k \in \mathbb{Z},\ b = ka$, we write $a \mid b$,
for $a, b \in \mathbb{Z}$.

$\forall n \in \mathbb{N},\ \underbrace{5 \mid n} \Rightarrow 5 \mid (n+5)$

$\forall n \in \mathbb{N},\ [\exists k_1 \in \mathbb{Z},\ n = 5k_1] \Rightarrow [\exists k_2 \in \mathbb{Z},\ n + 5 = 5k_2]$

Prime(n): $n > 1 \wedge \forall d \in \mathbb{N},\ \underbrace{d \mid n} \Rightarrow d = 1 \vee d = n$,
for $n \in \mathbb{N}$

same as $n > 1 \wedge \forall d \in \mathbb{N},\ [\exists k \in \mathbb{Z},\ n = dk] \Rightarrow d = 1 \vee d = n$

restrict the domain

given domain U with $P, Q \subseteq U$

$P(x) : x \in P$, for $x \in U$ and $Q(x) : x \in Q$, for $x \in U$

e.g. $P(x) : x > 17$

$Q(x) : x > 3$

$$\exists x \in P, Q(x) \iff \exists x \in U, x \in P \wedge Q$$

$$\iff \exists x \in U, \underbrace{P(x) \wedge Q(x)}$$

not same as $P(x) \Rightarrow Q(x)$

$$\forall x \in P, Q(x) \iff P \subseteq Q$$

$$\iff \forall x \in U, P(x) \Rightarrow Q(x)$$

not same as $P(x) \wedge Q(x)$

prove $3 \mid 18$

$\exists k \in \mathbb{Z},\ 18 = 3k$

<u>discuss</u> $3 \times 6 = 18$, so 6 "will work."

<u>Proof</u> Let $k = 6$. WTS $3k = 18$.

$3k = 3 \times 6 = 18$ ■

There is a real solution to $x^2 + 2x + 3 = 11$

$\in \mathbb{R}$

$\exists k \in \mathbb{R},\ k^2 + 2k + 3 = 11$

discussion True because 2 satisfies

Proof Let $x = 2$. WTS $x^2 + 2x + 3 = 11$.

$$2^2 + 2 \cdot 2 + 3 = x^2 + 2x + 3$$
$$= 8 + 3$$
$$= 11$$

∎

prove $n^2 + 2n + 5 > 4$ if $n \in \mathbb{N}$

$n \in \mathbb{N} \Rightarrow n^2 + 2n + 5 > 4$

eqv $\forall n \in \mathbb{N}, n^2 + 2n + 5 > 4$

discuss 5 is notoriously > 4, so adding some non-negative terms preserves this.

Proof Let n be a fixed, arbitrary natural number. WTS $n^2 + 2n + 5 > 4$.

$n \geq 0$ # $n \in \mathbb{N}$

$n^2 \geq 0$

$n^2 + n \geq n \geq 0$

$n^2 + 2n \geq n + n \geq 0$

$n^2 + 2n + 5 \geq 5 > 4$ $\blacksquare$