CSC165 fall 2019

Mathematical expression: predicate logic

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Using Course notes: Mathematical Expression: predicate logic

Outline

implication with predicates

quantifiers

multiple quantifiers

negation

number theory intro

notes

the venn of implication...

$p(x)$: "$x > 4$", for $x \in \mathbb{R}$
$q(x)$: "$x > 3$", for $x \in \mathbb{R}$
$f(x)$ "$x^2 + 1 = 0$", for $x \in \mathbb{R}$
define $P = \{x : x \in \mathbb{R} \wedge p(x)\}$, $Q = \{x : x \in \mathbb{R} \wedge q(x)\}$, $F = \{x : x \in \mathbb{R} \wedge f(x)\}$
draw Venn diagrams of P, Q, F and compare to $p(x) \Rightarrow q(x), f(x) \Rightarrow q(x)$

more venn of implication...

$p(x)$: "$x > 4$", for $x \in \mathbb{R}$
$q(x)$: "$x > 3$", for $x \in \mathbb{R}$
$f(x)$ "$x^2 + 1 = 0$", for $x \in \mathbb{R}$
define $P = \{x : x \in \mathbb{R} \wedge p(x)\}$, $Q = \{x : x \in \mathbb{R} \wedge q(x)\}$, $F = \{x : x \in \mathbb{R} \wedge f(x)\}$
draw Venn diagrams of P^c, Q^c, F^c and compare to $\neg q(x) \Rightarrow \neg p(x), \neg q(x) \Rightarrow \neg f(x)$

quantifiers $\forall$ and $\exists$

$f(n)$: "$n > 7$", for $n \in \mathbb{N}$

translate quantified predicates

multiple matched quantifiers...

$A = \{1, 3, 5\}, B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\forall x \in A, \forall y \in B, p(x, y)$ versus $\exists x \in A, \exists y \in B, p(x, y)$

multiple matched quantifiers continued...

$A = \{1, 3, 5\}, B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\forall x \in A, \forall y \in B, p(x, y)$ versus $\exists x \in A, \exists y \in B, p(x, y)$

mixed quantifiers

$A = \{1, 3, 5\}, B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\forall x \in A, \exists y \in B, p(x, y)$ versus $\exists y \in B, \forall x \in A, p(x, y)$

mixed quantifiers continued...

$A = \{1, 3, 5\}$, $B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\forall x \in A, \exists y \in B, p(x, y)$ versus $\exists y \in B, \forall x \in A, p(x, y)$

$\exists$: examples

$\forall$: lack of counterexamples

negate $\neg$, $\wedge$, $\vee$

negate quantified predicates

manipulate negation

$\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x \geq 5 \vee x^2 - y \geq 30$

properties of integers, mostly $\mathbb{N}$

divisibility

primes

Notes