PS0: due tomorrow @ 4
quiz 3: due Thurs @ 1

- late enrolling & missed work?

# CSC165 fall 2019

## Mathematical expression: predicate logic


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Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: Mathematical Expression: predicate logic

# multiple matched quantifiers...

$A = \{1, 3, 5\}$, $B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\underbrace{\forall x \in A, \forall y \in B}$, $p(x, y)$ versus $\underbrace{\exists x \in A, \exists y \in B}$, $p(x, y)$

swap without changing meaning

# mixed quantifiers

$A = \{1, 3, 5\}$, $B = \{6, 8, 10\}$

$p(x, y)$: "$x + y = 11$", for $(x, y) \in A \times B$

$\underline{\forall x \in A, \exists y} \in B, p(x, y)$ versus $\underline{\exists y \in B, \forall x} \in A, p(x, y)$

swapping (can) change meaning!

$\exists$: examples — just one example!

$\forall$: lack of counterexamples — no counterexamples negate $\neg, \wedge, \vee$ $\equiv$ equivalent

$$\neg\neg P(x) \equiv P(x)$$

De Morgan's Law

$$\neg(P(x) \wedge Q(x)) \equiv \neg P(x) \vee \neg Q(x)$$

$$\neg(Q(x) \vee P(x)) \equiv \neg Q(x) \wedge \neg P(x)$$

$=, >, <$

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$\neg(x = y) \equiv x \neq y$ $(x != y)$

$\neg(x < y) \equiv x \geq y$ OR $x \nless y$

$\neg(x \leq y) \equiv x > y$

# negate quantified predicates

$$\neg(\forall x \in X, P(x)) \equiv \exists x \in X, \neg P(x)$$

$$\neg(\exists y \in Y, Q(y)) \equiv \forall y \in Y, \neg Q(y)$$

# manipulate negation

$$\neg(P(x) \Rightarrow Q(x)) \equiv \neg(\neg P(x) \vee Q(x))$$

$$\equiv \neg\neg P(x) \wedge \neg Q(x)$$

$$\equiv P(x) \wedge \neg Q(x)$$

$$\neg(P(x) \Leftrightarrow Q(x)) \equiv [P(x) \wedge \neg Q(x)] \vee [\neg P(x) \wedge Q(x)]$$

$$\neg\left(\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x \geq 5 \vee x^2 - y \geq 30\right)$$

$$\equiv \exists x \in \mathbb{N}, \neg \ldots$$

$$\equiv \exists x \in \mathbb{N}, \forall y \in \mathbb{N}, \neg \ldots$$

$$\equiv \exists x \in \mathbb{N}, \forall y \in \mathbb{N}, x < 5 \wedge x^2 - y < 30$$

$$\cancel{\overset{?}{\equiv}}\ \cancel{\exists x \in \mathbb{N}, \exists y \in \mathbb{N}, x < 5 \wedge x^2 - y < 30}$$

# properties of integers, mostly $\mathbb{N}$

Number Theory

(want more → MAT 315)

- cryptography
- neat results

# divisibility

do not confuse with numerical divides or $\frac{x}{y}$ ✗✗✗

divides$(d, n)$: $\exists k \in \mathbb{Z}, n = kd$, for $d, n \in \mathbb{Z}$.

use, (write) operator $|$, i.e. $d \mid n$

$\forall n \in \mathbb{Z},\ n \mid 10 \Rightarrow n \mid 1000$

# primes

$\text{Prime}(n): n > 1 \wedge \forall d \in \mathbb{N}, d \mid n \Rightarrow d = 1 \vee d = n,$

for $n \in \mathbb{N}$.

- infinitely many (proven 2500 years ago)
- no obvious pattern