CSC165 fall 2019

graph connectivity

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Using Course notes: graphs
must be connected

∀n ∈ ℕ, ∃M ∈ ℕ, ∀G = (V, E), (|V| = n ∧ |E| ≥ M) ⇒ G is connected?
must be connected

∀n ∈ ℕ, ∃M ∈ ℕ, ∀G = (V, E), (|V| = n ∧ |E| ≥ M) ⇒ G is connected?
must be connected

\[ \forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (|V| = n \land |E| \geq M) \Rightarrow G \text{ is connected?} \]
maybe connected

\[ \forall n \in \mathbb{N}, \exists G = (V, E), |V| = n \land |E| = n - 1 \land G \text{ is connected} \]
must be disconnected

\( \forall n \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \leq n - 2) \Rightarrow G \) is not connected

steps:

- natural to reason by removing an edge from a connected graph with \( n - 1 \) edges...
- first need some results about which components of connected graphs have redundant edges (cycles)...
- then need some results about connected graphs without cycles (trees)...
- then reason about reducing an arbitrary connected graph to a tree...

whew!
cycle
consecutively adjacent vertices $v_0, \ldots, v_k \in V \land k \geq 3,$
all distinct except $v_0 = v_k$
$\forall G = (V, E), \forall e \in E, G$ connected $\Rightarrow (e$ in a cycle of $G \iff G - e$ connected $)$
tree: connected, acyclic graph

removing any edge from a tree disconnects it

\[ \forall G = (V, E), \text{ } G \text{ is a tree } \Rightarrow |E| = |V| - 1 \]

but first...

\[ \forall G = (V, E), (G \text{ is a tree } \wedge |V| \geq 2) \Rightarrow (\exists v \in V, d(v) = 1) \]
main result...

\[ \forall n \in \mathbb{N}^+, \forall G = (V, E), (G \text{ is a tree } \land |V| = n) \Rightarrow |E| = |V| - 1 \]
big picture...