CSC165 fall 2019

graph connectivity

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Using Course notes: graphs

must be connected

$\forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \geq M) \Rightarrow G$ is connected?

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maybe connected

$\forall n \in \mathbb{N}, \exists G = (V, E), |V| = n \wedge |E| = n - 1 \wedge G$ is connected

must be disconnected

$\forall n \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \leq n - 2) \Rightarrow G$ is not connected

steps:

- natural to reason by removing an edge from a connected graph with $n - 1$ edges...
- first need some results about which components of connected graphs have redundant edges (cycles)...
- then need some results about connected graphs without cycles (trees)...
- then reason about reducing an arbitrary connected graph to a tree...

whew!

cycle

consecutively adjacent vertices $v_0, \ldots, v_k \in V \wedge k \geq 3$,
all distinct except $v_0 = v_k$

$\forall G = (V, E), \forall e \in E, G \text{ connected } \Rightarrow (e \text{ in a cycle of} G \Leftrightarrow G - e \text{ connected })$

tree: connected, acyclic graph

removing any edge from a tree disconnects it

$\forall G = (V, E), G \text{ is a tree } \Rightarrow |E| = |V| - 1$

but first...

$\forall G = (V, E), (G \text{ is a tree } \wedge |V| \geq 2) \Rightarrow (\exists v \in V, d(v) = 1)$

main result...

$\forall n \in \mathbb{N}^+, \forall G = (V, E), (G \text{ is a tree } \wedge |V| = n) \Rightarrow |E| = |V| - 1$

big picture...

Notes