CSC165 fall 2019

rooted trees / what's next

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Using Course notes: trees

distinguish a root

add notions of distance, hierarchy/direction to trees by

rooted tree: a tree with

- exactly one vertex labelled (distinguished) as root, if the tree has at least one vertex
- OR no vertices (a convenience for proofs and algorithms)

jargon

- parent
- child
- ancestor
- descendant
- arity (branching factor)
- height, denote as $height(G)$

easy-ish facts

- every rooted tree with $n \geq 2$ vertices has height at least 2
- some rooted tree with $n \geq 2$ vertices has height exactly 2
- every rooted tree with n vertices has height no more than n
- some rooted tree with n vertices has height exactly n

binary rooted trees

maximum degree 3 $\equiv$ maximum of 2 children

$\forall h \in \mathbb{N}, \forall G = (V, E)\,(G \text{ rooted, binary tree } \wedge \mathit{height}(G) \leq h) \Rightarrow |V| \leq 2^h - 1$

later topics...

- prove correctness
- analyze recursive runtime
- computability
- intractability
- public-key cryptography

problem with keys... e.g. Vigenere cipher

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

```
key:       thewalrusandthecarpenter
cleartext: ifsevenmaidswithsevenmopssweptforhalfayear

ifsevenmaidswithsevenmopssweptforhalfayear
thewalrusandthecarpenterthewalrusandthecar
```

how do you securely exchange keys?

public/private

share public key with the world
keep private key secret

allows:

authentication

encryption

RSA

need: text→integer, integer→text reversible padding scheme

1. randomly choose **large** primes p and q
2. $n = pq$ (key length is n in bits...)
3. $L = (p-1)(q-1)$
4. choose $1 < e < L$ so that $\gcd(e, L) = 1$
5. compute inverse, $d \equiv e^{-1} \pmod{L}$, i.e. $de \equiv 1 \pmod{L}$ (notes Example 2.19 works for **co-prime!**)

publish: e, n
keep private d, p, q, L.
$m = \text{text} \rightarrow \text{integer}(\text{message})$
encrypt: $c \equiv m^e \pmod{n}$
decrypt: $\text{message} = \text{integer} \rightarrow \text{text}(c^d) \pmod{n})$

it works... how?

Use results from this course... mostly

- $c^d \equiv m^{ed} \pmod{n}$
- $n = pq$, and $ed \equiv 1 \pmod{(p-1)(q-1)}$, i.e. $ed = 1 + k(p-1)(q-1)$
- $m^{ed} \equiv m \times m^{(p-1)(q-1)k} \pmod{p} \equiv m \times 1^{(q-1)k} \pmod{p}$ (problem set #3, Q1(c) almost...) $\equiv m \pmod{p}$
- also $m^{ed} \equiv m \pmod{q}$
- (problem set #2, Q2(a)): $m^{ed} \equiv m \pmod{pq} \equiv m \pmod{n}$.

Notes