No more worksheets... :(— but TAs will answer questions on recent ones...

Also FE 2017, on web site, out of 64 points

FE 2019 (not posted) will be out of 60 points ... 3 min/point ...

CSC165 fall 2019

graph connectivity

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Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: graphs

must be connected

$|V| = n$

$\frac{(n-1)(n-2)}{2} + 1$

$\forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \geq M) \Rightarrow G$ is connected?

maybe connected

$\forall n \in \mathbb{N}, \exists G = (V, E), |V| = n \wedge |E| = n - 1 \wedge G$ is connected

$\exists$ path P with n vertice and $n-1$ edges. Ex Show P is connected.

choose 2 vertices in P, show there is a path.

$|V| \cong 2.41B$ could

create P with $2.41B - 1$ edges.

$$\frac{(2.41B-1)(2.41B-2)}{2} + 1$$

must be disconnected

$\forall n \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \leq n - 2) \Rightarrow G$ is not connected

steps:

- natural to reason by removing an edge from a connected graph with $n - 1$ edges...
- first need some results about which components of connected graphs have redundant edges (cycles)...
- then need some results about connected graphs without cycles (trees)...
- then reason about reducing an arbitrary connected graph to a tree...

whew!

cycle

$G=(V,E)$

$G-e = (V, E-\{e\})$

consecutively adjacent vertices $v_0, \ldots, v_k \in V \wedge k \geq 3$, all distinct except $v_0 = v_k$

$\forall G=(V,E), \forall e \in E, G$ connected $\Rightarrow$ (e in a cycle of $G \Leftrightarrow G-e$ connected)

Proof $\Longrightarrow$ Let $G=(V,E)$ be a conn graph and $e \in E$ be an edge in some cycle in G. WTS That $G' = G-e$ is also connected. Let $w_1, w_2 \in V$. By assumption, $\exists$ path $P = w_1, \ldots, w_2$. in G. In G' there are 2 cases.

Case 1 e is not in P. Then P is also a path in G', so w_1 is connected to w_2 in G'.

Case 2 $e = (u,v)$ is in P. WLOG, assume that u is closer to w_1 than v is in P.

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Then $w_1, \dots, u$ is part of P that connects w_1 to u in G', and $v, \dots, w_2$ is part of P and connects v to w_2 in G'.

Thanks to clever boffins working on PS4, u is connected to v in G'. So, by transitivity, w_1 is connected to v, and w_1 (trans again) is connected to w_2 $\blacksquare$

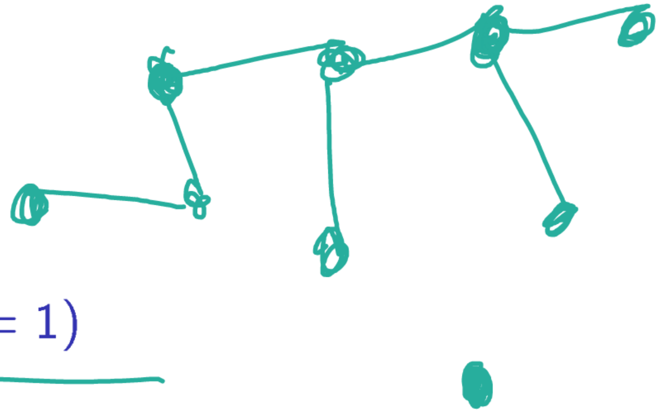
$$P - e = w_1, \dots, u$$
$$v, \dots, w_2$$

Proof $\Leftarrow$

Let $G = (V, E)$. Assume G connected and $\exists\, e \in E$, $G - e = G'$ is connected. WTS $\exists$ cycle C in G with e an edge in C.

If $e = (u,v)$, then since G' is connected, there is a path $u, \ldots, v$ connecting u to v in G'. This path has length > 1, since the edge from u to v was removed. But then $u, \ldots, v, u$ is a cycle in G containing e ∎

tree: connected, acyclic graph



removing any edge from a tree disconnects it

$\forall G = (V, E), G \text{ is a tree } \Rightarrow |E| = |V| - 1$

but first...

$\forall G = (V, E), (G \text{ is a tree } \wedge |V| \geq 2) \Rightarrow (\exists v \in V, d(v) = 1)$

Exercise proved in notes.

main result...

$\forall n \in \mathbb{N}^+, \forall G = (V, E), (G \text{ is a tree } \wedge |V| = n) \Rightarrow |E| = |V| - 1$

cannot be connected

$n-1$ edges

?

$\frac{(n-1)(n-2)}{2} + 1$ edges

must be connected

big picture...

Notes