t2: average/mean too low... what to do?

- time pressure
- concepts → read the PS solutions

fe ≤ 2 x t2 in length...
but 180 minutes

CSC165 fall 2019

graph connectivity

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http://www.teach.cs.toronto.edu/~heap/165/F19/

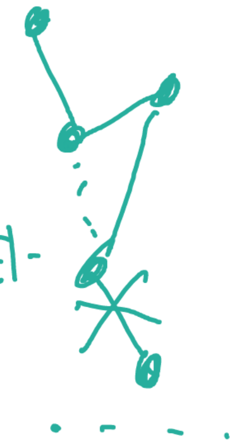
Using Course notes: graphs

must be connected

$\frac{n(n-1)}{2} = M$ ✓ Can we do better?

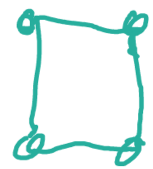
$\forall n \in \mathbb{N}, \exists M \in \mathbb{N}, \forall G = (V, E), (|V| = n \wedge |E| \geq M) \Rightarrow G$ is connected?

$P(n): \forall G = (V,E), [|V| = n \wedge |E| \geq \frac{(n-1)(n-2)}{2} + 1 \Rightarrow G$ connected]



Prove $\forall n \in \mathbb{N}^+, P(n)$

base case $P(1)$: Vacuously true.



Inductive step Let $n \in \mathbb{N}^+$. Assume $P(n)$

Induction trap: don't do this!

$\cancel{M} = n-1$

take an arb. graph with n vertices & $\geq \frac{(n-1)(n-2)}{2} + 1$ edges + $\cancel{M} = \frac{(n-1)(n-2)}{2}$

extend to graph with 1 more vertex + "enough" edges this gives an <u>example</u> that satisfies $P(n+1)$, but doesn't show for all.

<u>don't</u> !!!

<u>instead</u> Let G $= (V, E)$ be an arb. graph with $|V| = n+1$ and $|E| \geq \frac{n(n-1)}{2} + 1$ edges. WTS G connected. Want $v \in V$ with $1 \leq d(v) \leq n-1$

<u>Case 1:</u> $|E| = \frac{n(n+1)}{2}$, G is complete graph and is connected.

<u>Case 2:</u> $|E| < \frac{n(n+1)}{2} \wedge |E| \geq \frac{n(n-1)}{2} + 1$. Then $\exists v \in V$, $1 \leq d(v) \leq n-1$. Let $G' = (V', E')$, where $V' = V \setminus \{v\}$ and $E' = E - \underbrace{\{(u,v) : (u,v) \in E\}}_{\leq n-1}$

Since $|V'| = n \wedge |E'| \geq \frac{n(n-1)}{2} + 1 - (n-1)$

$$= \frac{n(n-1) - 2(n-1)}{2} + 1$$

$$= \frac{(n-2)(n-1)}{2} + 1$$

By IH, G' is connected.

also, v has at least one neighbour u, and u is connected to all of G' so v is (transitivity) connected to all of G'. So G is connected. $\blacksquare$