office hours: 4, tomorrow 1-4
VL vs exact: remember laziness

CSC165 fall 2019
average/summation/graphs

Danny Heap
csc165-2019-09@cs.toronto.edu
BA4270 (behind elevators)
Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: average analysis; graphs
finding a needle...
...when you know it's in the haystack

assume uniform dist.

# num_list is a list of numbers, # a permutation of {1, 2, 3, ..., n}
def find_one(num_list):
    for i in range(len(num_list)):
        if num_list[i] == 1:
            return i

\[
\frac{1}{n!} \sum_{i=1}^{n} i (n-1)! = \frac{(n-1)!}{n!} \sum_{i=1}^{n} \frac{i}{n!} \quad \text{as } \frac{1}{(n-1)!} \to 1
\]

\[
\approx \frac{1}{n} \sum_{i=1}^{n} i = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2}
\]
graphs (discrete ones)...
what can you do with them?

- represent friendships
  \[ V = \frac{1}{2} \text{Facebook accounts} \]
  \[ E = \frac{1}{2} \text{friendships from FB} \]

- represent lecture sections
  \[ V = \text{lecture sections at UofT} \]
  \[ E = \{ (c_1, c_2) : c_1, c_2 \in V \land c_1, c_2 \text{ share students} \} \]

- represent tasks\(\leftrightarrow\)person
  \[ V = \{ \text{people} V \text{ jobs} \} \]
  \[ E = \{ (p, j) : p, j \in V \land p \text{ qualified for } j \} \]
Definitions...

$G = (V, E) \in \mathcal{G}$

set of all graphs that simple and finite

simple no more than 1 edge per pair

forbidden $X \cdots X$ no!

no self loops $\cdots X$ no!

$|V| \in \mathbb{N}$

given this suppose $|V| = n$ (for some $G = (V, E)$)

and $|E| = m$ (for some $G = (V, E)$)

$0 \leq m \leq \frac{n(n-1)}{2} - \text{max choices for } (u, v)$
degree, degree-sum, max number of edges?

\[ d(v) : \sum \frac{1}{2} (v, u) : (v, u) \in E \] for \( G = (V, E) \)

\[ d(v) = 2 \]
\[ d(w) = d(x) = 1 \]

\[ \text{degree-sum } (G = (V, E)) = \sum_{v \in V} d(v) \]

\[ G = \begin{array}{c}
\text{degree-sum}(G) = 12
\end{array} \]

\[ G' = \begin{array}{c}
d-s(G') = 8
\end{array} \]
paths, connectedness... in $G = (V, E)$

A path from $u$ to $v$: Distinct vertices $v_0, \ldots, v_k$ in $V$ where:

- $u = v_0, v = v_k$
- if $0 \leq i \leq k - 1$, then $(v_i, v_{i+1}) \in E$

We allow $k = 0$ — there is a path from $v$ to itself.

Path length from $u$ to $v$: number of edges in path from $u$ to $v$.

$u, v$ are connected: There is a path from $u$ to $v$.

Graph $G$ is connected: every pair $u, v \in G$ is connected.