office hour(s): 4, tomorrow 1-4

U/L vs exact: remember laziness

varying: $\lceil \frac{n}{6} \rceil$ ✓

ps3 - solutions

ps4:

CSC165 fall 2019

average/summation/graphs

Danny Heap

csc165-2019-09@cs.toronto.edu

BA4270 (behind elevators)

Web page:

http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: average analysis; graphs

finding a needle...

...when you know it's in the haystack

assume uniform dist.

$$\left| I_{find\_one,n} \right| = n!$$

$$\frac{1}{n!} \sum_{x \in I_{find\_one,n}} RT_{find\_one,n}(x)$$

```
# num_list is a list of  numbers,
# a permutation of {1, 2, 3, ..., n}
def find_one(num_list):
    for i in range(len(num_list)):
        if num_list[i] == 1:
            return i
```

how many lists have 1 in first pos? $(n-1)!$

" " as pos i? $(n-1)!$

$$\frac{1}{n!} \sum_{i=1}^{n} i(n-1)! = \frac{(n-1)!}{n!} \sum_{i=1}^{n} i$$

$$= \frac{1}{n} \sum_{i=1}^{n} i = \frac{1}{n} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2} \quad //$$

graphs (discrete ones)...

what can you do with them?

$G = (V, E)$

V - some finite set of vertices

E - some pairs from V

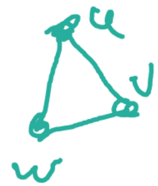
$|V| = 2.4B$

$u, v, w \in V$

- represent friendships

$V = \{\text{facebook accounts}\}$

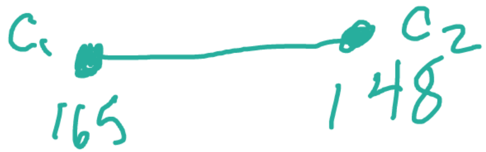
$E = \{\text{"friend"ships from FB}\}$



- represent lecture sections

$V = \{\text{lecture sections @ UofT}\}$

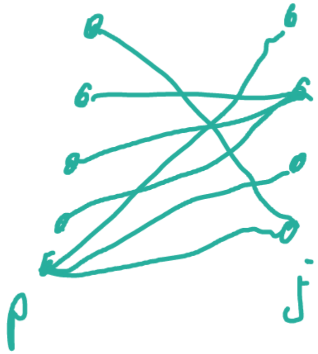
$E = \{(c_1, c_2) : c_1, c_2 \in V \wedge c_1, c_2 \text{ share students}\}$



- represent tasks↔person

$V = \{\text{people} \vee \text{jobs}\}$

$E\ \{(p, j) : p, j \in V \wedge p \text{ qualified for } j\}$



bi-partitions

definitions...

$G = (V, E) \in \mathcal{G}$

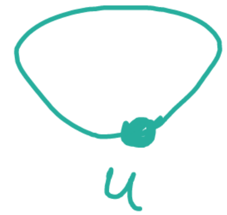
Set of all graphs that simple + finite

simple no more than 1 edge per pair

forbidden

X no!

no self loops



X no!

$|V| \in \mathbb{N}$

given this suppose $|V| = n$ and $|E| = m$ (for some $G=(V,E)$)

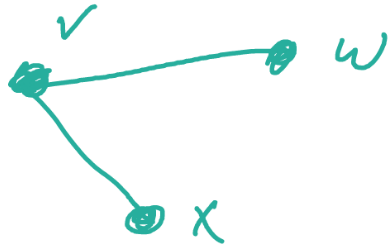
$0 \le m \le \frac{n(n-1)}{2}$ — max choices for (u,v)

degree, degree-sum, max number of edges?

$d(v) : |\{(v,u) : (v,u) \in E\}| \ \ldots$ for $G = (V,E)$

$d(u) = 3$

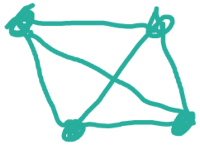
d



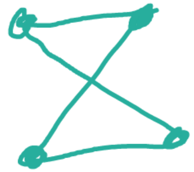
$d(v) = 2$

$d(w) = d(x) = 1$

$$\text{degree-sum}(G = (V,E)) = \sum_{v \in V} d(v)$$

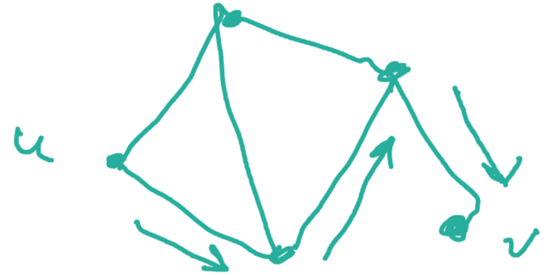
$G =$  — C_4

degree-sum$(G) = 12$

$G' =$ 

d-s$(G') = 8$

paths, connectedness... in $G = (V, E)$



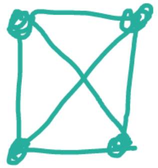
A path from u to v: Distinct vertices $v_0, \ldots, v_k$ in V where:

- $u = v_0$, $v = v_k$
- if $0 \leq i \leq k - 1$, then $(v_i, v_{i+1}) \in E$

We allow $k = 0$ — there is a path from v to itself

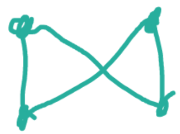
path length from u to v: number of edges in path from u to v

connect!



u, v are connected: There is a path from u to v.

graph G is connected: every pair $u, v \in G$ is connected



connected