CSC165 fall 2019
Mathematical expression

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Web page:
http://www.teach.cs.toronto.edu/~heap/165/F19/

Using Course notes: Prologue, Mathematical Expression
Outline

Introduction

sets

functions

sums and products

propositional logic

notes
what’s CSC165?
a course about expression (communication):

- with and through programs
- with developers
- knowing what you mean
- understanding what others mean
- analyzing arguments, programs
- understanding cool domains (number theory, graphs, etc.)
CS needs math:

- geometry, linear algebra
- graphics
- logic
- number theory, RSA
- cryptography
- probability
- artificial intelligence
- efficiency, algebra
- complexity
- numerical analysis, calculus
- networking, stats
- databases, set theory

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by the end of this course you will able to

- express mathematical ideas precisely
- read and understand other people’s proofs
- read and identify flaws in incorrect proofs
- express your own proofs
- analyze (some) program complexity
- engage with number theory, graph theory
Doing well in CSC165

Doing well has two aspects: one being recognized as doing well by being awarded credit (grades), another being able to retain concepts and tools for use later on. Here’s how to do both:

▸ build a network of good peers

▸ read the course web page, and emails, regularly; understand the course information sheet.

▸ spend enough time; we assume an average of 8 hours/week — 4 in lecture/problem sessions, 4 reviewing preparing assignments

▸ ask questions; make your own annotations.
typical week workflow

- reading and prep — do your quiz!
- lectures — ask questions!
- work sheet(s) — talk to TAs!
- problem sets — start early! — several sleeps!
- tests — study groups!

NB: This exam for this course is based on... this course! The best preparation is re-working all the materials listed above not old exams...
balance

no sense of humor

gm computers are precise — in identical environments they execute identical instructions identically

gm humans are as precise as necessary, and different human audiences require different levels of precision

gm The really difficult job is finding the right level of precision. Too much precision introduces unbearable tedium; too little introduces unfathomable ambiguity.

Proofs are primarily works of literature: they communicate with humans, and the best proofs have suspense, pathos, humour and surprise. As a side-effect, proofs present a convincing argument for some fact.
building sets... in math

All students in MP203 who have "i" as 2nd and 3rd letter of first name

\[ \{2, 3, 5, 7\} \]

English prose

list elements

\[ N = \{0, 1, 2, 3, 4, \ldots\} \]

set comprehension

\[ \{x : x \in N \text{ and } x > 15\} \]
some standard sets

\[ \mathbb{N} \text{ - see previous} \]
\[ \mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, 3, \ldots \} \]
\[ \mathbb{Q} = \{ \frac{p}{q} : p, q \in \mathbb{Z} \text{ and } q \neq 0 \} \]
\[ \mathbb{R} = (-\infty, +\infty) \]
Boolean operations on sets

\[ A = \{1, 3, 5\} \quad B = \{1, 3, 5, 7\} \]

- \(5 \in A \rightarrow \text{True}\)
- \(7 \in A \rightarrow \text{False}\)
- \(A \leq B \rightarrow \text{True}\)
- \(A \subseteq A \rightarrow \text{True}\)
- \(B \subseteq A \rightarrow \text{False}\)
- \(\emptyset \subseteq B \rightarrow \text{True}\)
- \(A = \{3, 5, 13\}\)
- \(\{1, 3, 5\} \subseteq \{1, 3, 5, 7\}\)
- \(A = B \rightarrow \text{False}\)
- \(A \subseteq B \text{ but } B \nsubseteq A\)

\[ \exists x \in \text{range}(10)\]
operations that produce new sets

\[ A = \{1, 3, 7, 5\} \]
\[ B = \{3, 5, 9, 11\} \]

\( \cap \) - intersection

\[ A \cap B = \{3, 5\} \]

\[ A \cup B = \{1, 3, 5, 7, 9, 11\} \]

\[ A \setminus B = A - B = \{1, 7\} \]

\[ A \setminus \emptyset = A \]

\[ A \cup \emptyset = A \]

\[ A \cap \emptyset = \emptyset \]
of sets... \[ |A| = 4 \]
\[ |B| = 4 \]
\[ |A \cap B| = 2 \]
\[ |A \cup B| = 6 \]
\[ |\mathbb{N}| = |\mathbb{Q}| = \infty \]
size of sets — cardinality

see previous slide
specify functions

\[ f: \{1, 3, 5\} \rightarrow \{T, F\} \]

\[ = \{ (1, T), (3, F), (5, T) \} \]

- ordered pairs

- pictures

- rule

\[ A = \{1, 3, 5\} \]
\[ B = \{2, 4, 6\} \]

\[ f: A \rightarrow B \]

\[ f(a) = a+1, \text{ for } a \in A \]
from/to domain/co-domain, arrow notation

\[ f : X \rightarrow Y \]

\[ \text{domain: source of inputs} \]
\[ \text{codomain: set containing outputs} \]
\[ \text{(may contain other things)} \]

\[ f : \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) = x + 3, \quad \text{for } x \in \mathbb{R} \]
one-to-one, onto, etc.

if \( x \neq y \), then \( f(x) \neq f(y) \) — onto

every \( x, y \in \text{domain} \)

**Example**

\( f: \mathbb{R} \rightarrow \mathbb{R} \).

\( f(x) = x^2 \)

there is

\( \exists \ x \in \mathbb{R} \text{ such that } f(x) = -5 \)

\( f(2) = f(-2) \rightarrow \text{not one-to-one} \)
Sums, products

\[
\sum_{i=0}^{100} 3i + i^2 = \sum_{i=0}^{100} 3i + \sum_{i=0}^{100} i^2 = 3 \sum_{i=0}^{100} i + \sum_{i=0}^{100} i^2
\]

\[
\sum_{i=1}^{9} i^2 = \sum_{i=1}^{9} (i+1)^2 - \sum_{i=0}^{8} i^2
\]

\[
\prod_{i=1}^{10} i(i+2) = (1 \cdot 2 \cdot 3 \ldots 10)(3 \cdot 4 \ldots 12) = (1 \cdot 3)(2 \cdot 4) \ldots (10 \cdot 12)
\]
manipulating sums and products
propositional logic

"Danny is tall" → False!

- statements, variables
  - stand for propositions
- operators
  - use p, q, r as variables

5 + 7

Not a proposition!
not \neg, \text{ and } \land

\begin{align*}
\neg p & | & \neg \neg p \\
T & | & F \\
F & | & T
\end{align*}

\begin{align*}
p & | & q & | & \land & | & \text{conjunction} \\
T & | & T & | & T & & \\
T & | & F & | & F & & \\
F & | & T & | & F & & \\
F & | & F & | & F & & \\
F & | & F & | & F & & \\
F & | & F & | & F & & \\
\end{align*}
or \( \lor \), implies \( \Rightarrow \)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>P \lor Q</th>
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<tbody>
<tr>
<td>T</td>
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Hypothesis

\[ p \lor q \]

Disjunction

\[ p \Rightarrow q \]

Conclusions

"Danny is tall"

If \( p \) then \( q \)

Vacuous