CSC165 fall 2019

Mathematical expression

face-to-face

Danny Heap

csc165-2019-09@cs.toronto.edu

BA4270 (behind elevators)

Web page:

http://www.teach.cs.toronto.edu/~heap/165/F19/

Most course info

Using Course notes: Prologue, Mathematical Expression

Outline

what's CSC165?

a course about expression (communication):

- with and through programs — expressing solutions to problems
- with developers — documentation
- knowing what **you** mean — could you explain it over the phone to a stranger
- understanding what **others** mean — read documentation specifications
- analyzing arguments, programs
  → correct, efficiency, elegance
- understanding cool domains (number theory, graphs, l..)

CS needs math:

- graphics — geometry, linear algebra
- verification — logic
- cryptography — number theory — RSA
- artificial intelligence — probability
- complexity — efficiency — algebra
- numerical analysis — calculus
- networking — stats
- databases — set theory

⋮

objectives

by the end of this course you will able to

- express mathematical ideas precisely — give you a language
- read and understand other people's proofs —
- read and identify flaws in incorrect proofs —
- express your own proofs ✓
- analyze (some) program complexity — big-Oh, big-Theta
- engage with number theory, graph theory

doing well in CSC165

post

Doing well has two aspects: one being recognized as doing well by being awarded credit (grades), another being able to retain concepts and tools for use later on. Here's how to do both:

- build a network of **good** peers
  respectful
  critical
- read the course web page, and emails, regularly; understand the course information sheet. back to this
- spend enough time; we assume an average of 8 hours/week — 4 in lecture/problem sessions, 4 reviewing preparing assignments
- ask questions; make your own annotations.

typical week workflow

- reading and prep — do your quiz!
- lectures — ask questions!
- work sheet(s) — talk to TAs!
- problem sets — start early! — several sleeps!
- tests — study groups!

NB: This exam for this course is based on... this course! The best preparation is re-working all the materials listed above **not** old exams...

balance

no sense of humor

- computers are precise — in identical environments they execute identical instructions identically

- humans are as precise as necessary, and different human audiences require different levels of precision

level of "typical" a csc 165 student

- The *really* difficult job is finding the right level of precision. Too much precision introduces unbearable tedium; too little introduces unfathomable ambiguity.

- Proofs are primarily works of literature: they communicate with humans, and the best proofs have suspense, pathos, humour and surprise. As a side-effect, proofs present a convincing argument for some fact.

building sets...in math

"all students in MP203 who have "g" as 2nd + 3rd letter of first name"

$= \{\}$, $\emptyset$

English prose

$\{1, 3, 5, 7\}$

list elements

$\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$

set comprehension

$\{x : x \in \mathbb{N} \text{ and } x > 15\}$

some standard sets

$\mathbb{N}$ – see previous

$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z} \text{ and } q \neq 0\}$

$\mathbb{R} = (-\infty, +\infty)$

boolean operations on sets

boolean $\longrightarrow \{True, False\}$

$A = \{1, 3, 5\}$ $B = \{1, 3, 5, 7\}$

$5 \in A \longrightarrow$ True

$7 \in A \longrightarrow$ False

$A \subseteq B \longrightarrow$ True

$A \subseteq A \longrightarrow$ True

$B \subseteq A \longrightarrow$ False

$\emptyset \subseteq B \longrightarrow$ True

$\longrightarrow A = \{3, 5, 1\}$

$A = B \longrightarrow$ False

$A \subseteq B$ but $B \nsubseteq A$

$\{1, 3, 5\}$

$\{x$ for x in range(10)$\}$

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operations that produce new sets

$A = \{1, 3, 7, 5\}$ $\qquad$ $B = \{3, 5, 9, 11\}$

$\cap$ - intersection

$A \cap B = \{3, 5\}$

$A \cup B = \{7, 5, 1, 3, 9, 11\}$

$A \setminus B = A - B = \{1, 7\}$

$A \setminus \emptyset = A$

$A \cup \emptyset = A$

$A \cap \emptyset = \emptyset$

~~sets~~ of sets. . .

size

$$|A| = 4$$

$$|B| = 4$$

$$|A \cap B| = 2$$

$$|A \cup B| = 6$$

connect these facts

$$|\mathbb{N}| = |\mathbb{Q}| = \infty$$

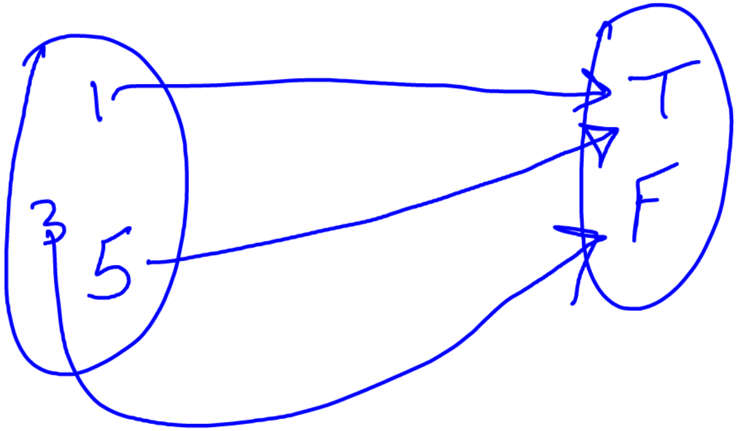
size of sets — cardinalty

see previous slide

specify functions

$$f : \{1, 3, 5\} \to \{T, F\}$$
$$= \{(1, T), (3, F), (5, T)\}$$

- ordered pairs
- pictures



- rule

$A = \{1, 5, 3\}$, $B = \{2, 4, 6\}$

$f : A \to B$

$f(a) = a + 1$, for $a \in A$

from/to, domain/co-domain, arrow notation

$f : X \longrightarrow Y$

~~domian~~ domain: source of inputs

codomain – set containing outputs

(may contain other things)

$f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(x) = x + 3$, for $x \in \mathbb{R}$

one-to-one, onto, etc.

if $x \neq y$, then $f(x) \neq f(y)$ — onto
every $x, y \in$ domain

<u>example</u> $f : \mathbb{R} \to \mathbb{R}$.

$f(x) = x^2$

there is <u>no</u> $x \in \mathbb{R}$ such that
$f(x) = -5$
$\longrightarrow$ not onto

$f(2) = f(-2) \longrightarrow$ not one-to-one

sums, products

$$\sum_{i=0}^{i=100} 3i + i^2 \quad \text{means} \quad 3\cdot 0 + 0^2 + 3\cdot 1 + 1^2 + 3\cdot 2 + 2^2 + \cdots + 3\cdot 100 + 100^2$$

$$= \sum_{i=0}^{i=100} 3i + \sum_{i=0}^{100} i^2 = 3\sum_{i=0}^{i=100} i + \sum_{i=0}^{i=100} i^2$$

$$\sum_{i=1}^{i=9} i^2 = \sum_{i=0}^{i=8} (i+1)^2 \qquad \ast \qquad \sum_{i=1}^{i=0} 3i = 0$$

$$\prod_{i=1}^{i=10} i\cdot(i+2) = (1\cdot 2\cdot 3 \cdots 10)(3\cdot 4 \cdots 12)$$

$$= (1\cdot 3)(2\cdot 4)\cdots(10\cdot 12)$$

$$\prod_{i=1}^{10} i \prod_{i=1}^{10} i+2$$

manipulating sums and products

propositional logic

"Danny is tall" $\longrightarrow$ False!

- statements, variables

stand for propositions

$\cancel{5+7}$ $\longleftarrow$ not a proposition!

- operators

– use p, q as variables

not $\neg$, and $\wedge$

p	$\neg p$
T	F
F	T

p	q	$\wedge$
T	T	T
T	F	F
F	T	F
F	F	F

conjunction!

or $\vee$, implies $\Rightarrow$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

hypothesis

disjunction

$[\neg p \vee q]$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

conclusion

"Danny is tall"

Vacuous

if p then q

Notes