Suppose we have a function f of two variables. At the point (x_0, y_0), what is the vector that points in the direction of steepest ascent?

As discussed in lecture, in order to increase f the most in the neighbourhood of (x_0, y_0), we need to move to

$$(x_0 + \alpha \frac{\partial f}{\partial x}(x_0, y_0) + \alpha \frac{\partial f}{\partial y}(x_0, y_0))$$

, for some α.

But where do you move along the z axis? Near (x_0, y_0), moving x by h moves $f(x, y)$ by $h\frac{\partial f}{\partial x}(x_0, y_0)$, and moving y by h movies $f(x, y)$ by $h\frac{\partial f}{\partial y}(x_0, y_0)$. We are doing both of those simoultaneously, and the result is moving f by

$$\alpha(\frac{\partial f}{\partial x}(x_0, y_0)^2 + \frac{\partial f}{\partial y}(x_0, y_0)^2)$$

.

The vector is therefore

$$(\frac{\partial f}{\partial x}(x_0, y_0), \frac{\partial f}{\partial y}(x_0, y_0), \frac{\partial f}{\partial x}(x_0, y_0)^2 + \frac{\partial f}{\partial y}(x_0, y_0)^2))^T.$$