CSC384 Final Review

Winter 2019

April 5, 2019
General Course Information

QUIZ
• Quiz marks have been posted to MarkUs.
• You can pick up your quiz in Bahen 3219 on weekdays, between 10-12 and 2-4pm.

Other Assignments
• A3 Remark Requests due Monday.
• A4 marks to be posted by Monday.
• Links to A4 will remain available for studying.
General Exam Information

We’ll be updating the CSC384 web page Test tab at the top of the page with any new exam information and also posting information on piazza.

Timing of Exam: Please look it up

Pre-Exam Help Sessions Proposal

- Monday, April 8, 1 - 3 pm (BA B026; Topic KR)
- Wednesday, April 10, 10 - 12 pm (BA5256; Topic CSPs)
- Wednesday, April 10, 12 - 1 pm (BA5256; Topic: CSPs)
- Wednesday, April 10, 3 - 5 pm (BA5256; Topic: KR)
- Thursday, April 11, 10 - 11 am (BA B024; Topic: Search)
- Thursday, April 11, 1 - 3 pm (BA B025; Topic: HMMs, BNs)
- Friday, April 12, 2 - 4 pm (BA B026; Topic: HMMs, BNs)

Exam Resources

- Old exams in library
- Problem sets from midterm etc.
- A4 problem sets online
- Brachman & Levesque KR book
- Russell & Norvig textbook
- Course slides and other posted materials

Any changes to times will be posted on piazza and on the “Test” web page
Tips and Resources

• Let the *lecture slides and the posted tutorial materials* be your guide for studying. If you’re unclear about something, augment it with the text or other online materials and/or come to an office hour!

• Make sure you *understand* the material. If there are proofs, work through them so you understand them. Understand the rationale for why things work the way they do.

• Work through some problem sets. Look at the posted sample problems on the test web page as well as problems we went through in class on the board, and old exams in the library.

• Know and understand the facts: know the complexity of different algorithms and why. Know the axioms of probability and understand how to apply.
General Exam Information

About the exam
• 3 hours in duration  
• no aids permitted  
• worth 40% of your course grade  
• You must get 40% on the exam to pass the course

Approximate distribution of marks on exam:
• Search(Game Tree, Uniformed, Informed): 4%  
• CSP: 20%  
• KR: 38%  
• Uncertainty: 38%
Today

→ KR Example

• CSP Example

• HMM Example
Resolution Example

One of Gordon Novak’s problems (posted on our Lectures page)

Consider the following axioms:

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (Conclusion) If John is a light sleeper, then John does not have any mice.

**TASK:** Prove the conclusion using resolution refutation
Resolution Example

Consider the following axioms:

1. All hounds howl at night.
2. Anyone who has any cats will not have any mice.
3. Light sleepers do not have anything which howls at night.
4. John has either a cat or a hound.
5. (Conclusion) If John is a light sleeper, then John does not have any mice.

The above English sentences can be written as the following first-order logic sentences.

1. \( \forall x \ (\text{HOUND}(x) \rightarrow \text{HOWL}(x)) \)
2. \( \forall x \ \forall y \ (\text{HAVE}(x,y) \land \text{CAT}(y) \rightarrow \neg \exists z \ (\text{HAVE}(x,z) \land \text{MOUSE}(z))) \)
3. \( \forall x \ (\text{LS}(x) \rightarrow \neg \exists y \ (\text{HAVE}(x,y) \land \text{HOWL}(y))) \)
4. \( \exists x \ (\text{HAVE}(\text{John},x) \land (\text{CAT}(x) \lor \text{HOUND}(x))) \)
5. \( \text{LS}(\text{John}) \rightarrow \neg \exists z \ (\text{HAVE}(\text{John},z) \land \text{MOUSE}(z)) \)
Resolution Example

1. $\forall x (\text{HOUND}(x) \rightarrow \text{HOWL}(x))$
2. $\forall x \forall y (\text{HAVE}(x,y) \land \text{CAT}(y) \rightarrow \neg \exists z (\text{HAVE}(x,z) \land \text{MOUSE}(z)))$
3. $\forall x (\text{LS}(x) \rightarrow \neg \exists y (\text{HAVE}(x,y) \land \text{HOWL}(y)))$
4. $\exists x (\text{HAVE}(\text{John},x) \land (\text{CAT}(x) \lor \text{HOUND}(x)))$
5. $\text{LS}(\text{John}) \rightarrow \neg \exists z (\text{HAVE}(\text{John},z) \land \text{MOUSE}(z))$

Convert to **Clausal Form** using the 8 steps we learned in class, yielding the following:

1. $\neg \text{Hound}(x), \text{Howl}(x))$
2. $\neg \text{Have}(x,y), \neg \text{Cat}(y), \neg \text{Have}(x,z), \neg \text{Mouse}(z))$
3. $\neg \text{LS}(x), \neg \text{Have}(x,y), \neg \text{Howl}(y))$
4. $\text{Have}(\text{John},a))$
5. $(\text{Cat}(a), \text{Hound}(a))$
6. $(\text{LS}(\text{John}))$
7. $(\text{Have}(\text{John},b))$
8. $(\text{Mouse}(b))$

Clause 1-3 correspond to Formulas 1-3, Clause 4&5 correspond to Formula 4, and Clauses 6-8 correspond to the **negation of Formula 5** (i.e., the negated query)
Resolution Example

Perform resolution refutation with these clauses to derive the empty clause:

1. \((\neg \text{Hound}(x), \text{Howl}(x))\)
2. \((\neg \text{Have}(x,y), \neg \text{Cat}(y), \neg \text{Have}(x,z), \neg \text{Mouse}(z))\)
3. \((\neg \text{LS}(x), \neg \text{Have}(x,y), \neg \text{Howl}(y))\)
4. \((\text{Have}(\text{John},a))\)
5. \((\text{Cat}(a), \text{Hound}(a))\)
6. \((\text{LS}(\text{John}))\)
7. \((\text{Have}(\text{John},b))\)
8. \((\text{Mouse}(b))\)

General Tips:

• Remember that each of these clauses is universally quantified from the outside using the variables that are contained in the clause. As such the “x” in clause 1 is different from the “x” in clauses, 2, 3, and 4. So if you’re resolving two clauses that each have a variable of the same name (e.g., “x”) after application of the MGU, you might wish to rename one of the “x”s to “z” or some other variable.

• A rule of thumb is to use the negated query or clauses derived from the negated query in your proof, since that negated query is the source of the inconsistency that will lead to the empty clause.
Resolution Example

Perform resolution refutation with these clauses to derive the empty clause:

1.  (¬Hound(x), Howl(x))
2.  (¬Have(x,y), ¬Cat(y), ¬Have(x,z), ¬Mouse(z))
3.  (¬LS(x), ¬Have(x,y), ¬Howl(y))
4.  (Have(John,a))
5.  (Cat(a), Hound(a))
6.  (LS(John))
7.  (Have(John,b))
8.  (Mouse(b))

We start the resolution with one of Clauses 6,7,8 which correspond to the negated query.

9.  R(2d,8a){z=b}    (¬Have(x,y), ¬Cat(y), ¬Have(x,b))
10. R(9c,7a){x=John} (¬Have(John,y), ¬Cat(y))
11. R(10b,5a){y=a}   (¬Have(John,a), Hound(a))
12. R(11b,1a){x=a}   (¬Have(John,a), Howl(a))
13. R(12b,3c){y=a}   (¬LS(x), ¬Have(x,a), ¬Have(John,a))
14. R(13a,6){x=John} (¬Have(John,a))
15. R(14,4)           () ←empty clause  QED
Today

- KR Example

→ CSP Example

- HMM Example
CSP Example

CSC384 Quiz-Test-Colour Assignment Problem

- 6 Students: Alice, Bob, Carol, Don, Ella, Fred …. {A,B,C,D,E}
- 3 different colours of quizzes: pink, green, blue …. {p,g,b}
- Students are sitting in seats in the following configuration:

Problem: Assign coloured quiz-tests to students so that students cannot look over and see another student’s quiz-test of the same colour.
CSP Example

- 6 Students: Alice, Bob, Carol, Don, Ella, Fred ....\{A, B, C, D, E\}
- 3 different colours of quizzes: pink, green, blue ....\{p, g, b\}

Constraints

A ≠ D, A ≠ C
B ≠ E, B ≠ F, B ≠ C
C ≠ D, C ≠ E, C ≠ F
D ≠ E

Initial Domains of Variables

Dom(A) = \{p, g, b\}
Dom(B) = \{p, g, b\}
Dom(C) = \{p, g, b\}
Dom(D) = \{p, g, b\}
Dom(E) = \{p, g, b\}
Dom(F) = \{p, g, b\}

Solve this CSP using Forward Checking.

Use a FIXED variable order: A, B, C, D, E, F
And a FIXED assignment order: p, g, b
Solve this CSP using **Forward Checking**.

Use a **FIXED** variable order: A,B,C,D,E,F
And a **FIXED** assignment order: p,g,b

Draw the search tree
At each node of the search tree indicate
- The variable being instantiated, and the value it is being assigned
- A list of the variables that have had at least one of their values pruned by the new assignment and a list of remaining legal values for each of these variables.
- Mark any DWOs
CSP Example

Draw the search tree.
Remember that you’re searching in the space of partial assignments to the variables. According to the fixed variable ordering defined in this problem (which we added to make the example illustrate some points), we start with variable A. We can assign A any value in its domain, i.e., any of p, g, or b. We start with p and then apply forward checking and in so doing prune the values in the domain of D and C to remove “p”. We proceed following the specified variable ordering.

A = p
Dom(D) = {g, b}, Dom(C) = {g, b}

B = p
Dom(E) = {g, b}, Dom(F) = {g, b}

B = g
Dom(E) = {p, b}, Dom(F) = {p, b}
Dom(C) = {b}

DWO
CSP Example

- Note that as variables are assigned, forward checking prunes the domains of variables.
- At the point of the first DWO, D only had b in its domain. As such the algorithm backtracks to C and tries the assignment C=b, since Dom(C)={g,b}.
- Observe that upon backtracking, the domains of variables are reinstated to their values at the level of backtracking.
Today

• KR Example

• CSP Example

→ HMM Example
We are flipping two coins (A and B).

Coin A comes up heads 70% of the time. Coin B comes up heads 40% of the time.

If we use coin A for a given toss, we will retain it for the next toss with probability of 0.4. If we use coin B for a given toss, the probability to retain coin B for the next toss is 0.8.

The coin initially flipped is equally likely to be coin A or coin B.

a) Formulate the experiment as a Hidden Markov Model.
b) What is the probability that we get "HT" on two successive tosses (H: Head; T: Tail)?
c) If we get "HT", what is the most likely sequence of coins (AA, AB, BA, or BB)?
We are flipping two coins (A and B).

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The coin initially flipped is equally likely to be coin A or coin B.

a) Formulate the experiment as a Hidden Markov Model.

```
Dom(S) = {A,B)
Dom(E) = {H,T}
P(S_1 = A) = 0.5
P(E=H|S = A) = 0.7
P(E=H|S = B) = 0.4
P(S_t=A|S_{t-1} = A) = 0.4
P(S_t=B|S_{t-1} = B) = 0.8
```
We are flipping two coins (A and B).

Coin A comes up heads 70% of the time. Coin B comes up heads 40% of the time.

If we use coin A for a given toss, we will retain it for the next toss with probability of 0.4.
If we use coin B for a given toss, the probability to retain coin B for the next toss is 0.8.

The coin initially flipped is equally likely to be coin A or coin B.

b) What is the probability that we get "HT" on two successive tosses (H: Head; T: Tail)?

\[
P(H,T) = P(S_2=A|S_1 = A)* P(S_1 = A)*P(E_1=H|S_1 = A)*P(E_2=T|S_2 = A) + \\
P(S_2=B|S_1 = A)* P(S_1 = A)*P(E_1=H|S_1 = A)*P(E_2=T|S_2 = B) + \\
P(S_2=A|S_1 = B)* P(S_1 = B)*P(E_1=H|S_1 = B)*P(E_2=T|S_2 = A) + \\
P(S_2=B|S_1 = B)* P(S_1 = B)*P(E_1=H|S_1 = B)*P(E_2=T|S_2 = B) = \\
0.4*0.5*0.7*0.3 + 0.6*0.5*0.7*0.6 + \\
0.2*0.5*0.4*0.3 + 0.8*0.5*0.4*0.6
\]

\[
\begin{align*}
\text{Dom}(S) &= \{A,B\} \\
\text{Dom}(E) &= \{H,T\} \\
P(S_1 = A) &= 0.5 \\
P(E=H|S = A) &= 0.7 \\
P(E=H|S = B) &= 0.4 \\
P(S_t=A|S_{t-1} = A) &= 0.4 \\
P(S_t=B|S_{t-1} = B) &= 0.8
\end{align*}
\]
HMM Example

We are flipping two coins (A and B).

Coin A comes up heads 70% of the time. Coin B comes up heads 40% of the time.

If we use coin A for a given toss, we will retain it for the next toss with probability of 0.4. If we use coin B for a given toss, the probability to retain coin B for the next toss is 0.8.

The coin initially flipped is equally likely to be coin A or coin B.

b) What is the probability that we get "HT" on two successive tosses (H: Head; T: Tail)?

Alternately, Using Bucket Notation:

\[ S_1: P(S_1), \ P(E_1=H|S_1), \ P(S_2|S_1) \]
\[ S_2: P(E_2=T|S_2) \]

\[ f_1(S_2): \]
\[ f_1(S_2 = A) = P(S_1=A)*P(E_1=H|S_1=A)*P(S_2=A|S_1=A) + P(S_1=B)*P(E_1=H|S_1=B)*P(S_2=A|S_1=B) = 0.5*0.7*0.4 + 0.5*0.4*0.2 \]
\[ f_1(S_2 = B) = P(S_1=A)*P(E_1=H|S_1=A)*P(S_2=B|S_1=A) + P(S_1=B)*P(E_1=H|S_1=B)*P(S_2=B|S_1=B) = 0.5*0.7*0.6 + 0.5*0.6*0.8 \]

\[ P(H,T) = P(E_2=T|S_2)*f_1(S_2) = 0.3* (0.5*0.7*0.4 + 0.5*0.4*0.2) + 0.6* (0.5*0.7*0.6 + 0.5*0.4*0.8) \]

Dom(S) = \{A,B\}
Dom(E) = \{H,T\}

P(S_1 = A) = 0.5
P(E=H|S = A) = 0.7
P(E=H|S = B) = 0.4
P(S_t=A|S_{t-1} = A) = 0.4
P(S_t=B|S_{t-1} = B) = 0.8
We are flipping two coins (A and B).

Coin A comes up heads 70% of the time. Coin B comes up heads 40% of the time.

If we use coin A for a given toss, we will retain it for the next toss with probability of 0.4. If we use coin B for a given toss, the probability to retain coin B for the next toss is 0.8.

The coin initially flipped is equally likely to be coin A or coin B.

c) If we get "HT", what is the most likely sequence of coins (AA, AB, BA, or BB)?

From Prior Solution We Learned:

\[
\begin{align*}
\text{Dom}(S) &= \{A, B\} \\
\text{Dom}(E) &= \{H, T\} \\
P(S_1 = A) &= 0.5 \\
P(E = H | S_1 = A) &= 0.7 \\
P(E = H | S_1 = B) &= 0.4 \\
P(S_2 = A | S_1 = A) &= 0.4 \times 0.5 \times 0.7 \times 0.3 = 0.042 \\
P(S_2 = B | S_1 = A) &= 0.6 \times 0.5 \times 0.7 \times 0.6 = 0.126 \\
P(S_2 = A | S_1 = B) &= 0.2 \times 0.5 \times 0.4 \times 0.3 = 0.012 \\
P(S_2 = B | S_1 = B) &= 0.8 \times 0.5 \times 0.4 \times 0.6 = 0.096 \\
\text{Biggest number above is 0.126, which corresponds to the sequence } A, B \\
P(S_t = A | S_{t-1} = A) &= 0.4 \\
P(S_t = B | S_{t-1} = B) &= 0.8
\end{align*}
\]