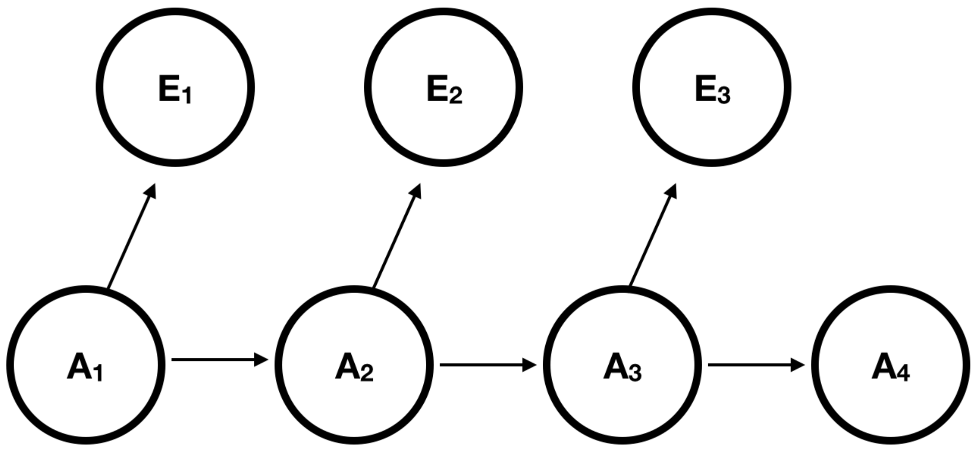
1 HMM Problem

1. Draw an HMM implied by the CPTs that are provided.



2. Calculate $P(A4 = true | E_1 = 0, E_2 = 1, E_3 = 0)$.

Solution:

$P(A4 = true | E_1 = 0, E_2 = 1, E_3 = 0) =$
$P(A4 = true, E_1 = 0, E_2 = 1, E_3 = 0)/P(E_1 = 0, E_2 = 1, E_3 = 0) =$
$\sum_{A1} \sum_{A2} \sum_{A3} P(A4 = true, A3, A2, A1, E_1 = 0, E_2 = 1, E_3 = 0)/$
$\sum_{A1} \sum_{A2} \sum_{A3} \sum_{A4} P(A4, A3, A2, A1, E_1 = 0, E_2 = 1, E_3 = 0)$

Variable elimination steps:

1. Restrict CPTs (i.e. eliminate/ignore values that are inconsistent with $E_1 = 0, E_2 = 1$, or $E_3 = 0$).

2. Eliminate A1, A2, A3.

Eliminate A1:

$f_1(A2) = \sum_{A1} P(A1)P(E_1 = 0|A1)P(A2|A1)$

$f_1(A2 = true) = P(A1 = true)P(E_1 = 0|A1 = true)P(A2 = true|A1 = true) + P(A1 = false)P(E_1 = 0|A1 = false)P(A2 = true|A1 = false))$
$= 0.99 * 0.8 * 0.99 + 0.01 * 0.1 * 0.01 = 0.784$
$f_1(A2 = false) = P(A1 = true)P(E_1 = 0|A1 = true)P(A2 = false|A1 = true) + P(A1 = false)P(E_1 = 0|A1 = false)P(A2 = false|A1 = false))$
$= 0.99 * 0.8 * 0.01 + 0.01 * 0.1 * 0.99 = 0.009$

Eliminate A2:

$f_2(A3) = \sum_{A2} P(E_2 = 1|A2)P(A3|A2)f_1(A2)$

$f_2(A3 = true) = P(E_2 = 1|A2 = true)P(A3 = true|A2 = true)f_1(A2 = true) + P(E_2 = 1|A2 = false)P(A3 = true|A2 = false)f_1(A2 = false)$
$= 0.2 * 0.99 * 0.784 + 0.9 * 0.01 * 0.009 = 0.155$
$f_2(A3 = false) = P(E_2 = 1|A2 = true)P(A3 = false|A2 = true)f_1(A2 = true) + P(E_2 = 1|A2 = false)P(A3 = false|A2 = false)f_1(A2 = false)$
$= 0.2 * 0.01 * 0.784 + 0.9 * 0.99 * 0.009 = 0.010$

Eliminate A3:

$f_3(A4) = \sum_{A3} P(E_3 = 0|A3)P(A4|A3)f_2(A3)$

$f_3(A4 = true) = P(E_3 = 0|A3 = true)P(A4 = true|A3 = true)f_2(A3 = true) + P(E_3 = 0|A3 = false)P(A4 = true|A3 = false)f_2(A3 = false)$
$= 0.8 * 0.99 * 0.155 + 0.1 * 0.01 * 0.010 = 0.123$
$f_3(A4 = false) = P(E_3 = 0|A3 = true)P(A4 = false|A3 = true)f_2(A3 = true) + P(E_3 = 0|A3 = false)P(A4 = false|A3 = false)f_2(A3 = false)$
$= 0.8 * 0.01 * 0.155 + 0.1 * 0.99 * 0.010 = 0.002$

3. Normalize.

$P(A4 = true|E_1 = 0, E_2 = 1, E_3 = 0) =$
$f_3(A4 = true)/(f_3(A4 = true) + f_3(A4 = false)) = .98$

3. What is the probability of observing the emission sequence $\{E_1 = 0, E_2 = 1, E_3 = 0\}$?

Solution:

Note that $P(E_1 = 0, E_2 = 1, E_3 = 0)$ is the normalizing factor in the elimination process above, i.e. $(f_3(A4 = true) + f_3(A4 = false)) = .125$

You can also calculate using much the same process as above (i.e. by leveraging the same factors):

$P(E_1 = 0, E_2 = 1, E_3 = 0) =$
$\sum_{A1} \sum_{A2} \sum_{A3} P(A3, A2, A1, E_1 = 0, E_2 = 1, E_3 = 0) =$
$\sum_{A1} \sum_{A2} \sum_{A3} = P(A3|A2)P(A2|A1)P(A1)P(E_1 = 0|A1)P(E_2 = 1|A2)P(E_3 = 0|A3) =$
$\sum_{A3} P(E_3 = 0|A3) \sum_{A2} P(A3|A2)P(E_2 = 1|A2) \sum_{A1} P(A1)P(A2|A1)P(E_1 = 0|A1) =$
$\sum_{A3} P(E_3 = 0|A3) \sum_{A2} P(A3|A2)P(E_2 = 1|A2)f_1(A2) = \sum_{A3} P(E_3 = 0|A3)f_2(A3) =$
$P(E_3 = 0|A3 = true)f_2(A3 = true) + P(E_3 = 0|A3 = false)f_2(A3 = false) = 0.155 * .8 + .01 * .1 = .125$