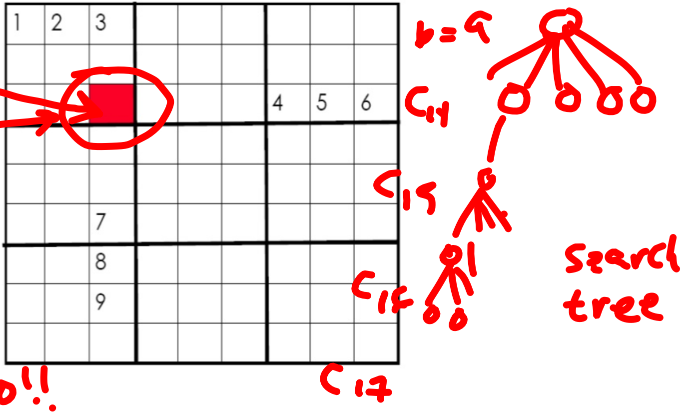
CSC384
Constraint Satisfaction Problems
Part 2

Bahar Aameri & Sonya Allin

Summer 2020

C_{33}

Not good way to find DWO!!



The backtracking search **won't** detect that the (3,3) cell has no possible value until all variables of the row/column/sub-square are assigned.

C_{33} down here!! → * C_{33} = 0 0 0 0 0 0 0 ...

Constraint Propagation

- In CSPs, there might be variables that have **no possible value**, but BT **doesn't detect** this until it tries to **assign** them a value.
  This leads to the idea of Constraint Propagation (or Domain Filtering).

Constraint Propagation: "looking ahead" at the yet unassigned variables in the search, trying to detect obvious failures.
"Obvious" means things we can test/detect efficiently.

- Even if it doesn't detect an obvious failure, it might be possible to **eliminate some parts** of the future search.

Constraint Propagation

- Propagation has to be applied **during the search**; potentially at **every node** of the search tree.
- Propagation itself is an inference step that needs some resources (in particular, **time**). If propagation is slow, this can slow the search down to the point where using propagation makes finding a solution take longer!
- Two main types of propagation: **Forward Checking** and **Generalized Arc Consistency**.

Constraint Propagation: Forward Checking

$C(X, Y, Z)$

$\rightarrow C(X=1, Y=1, Z)$

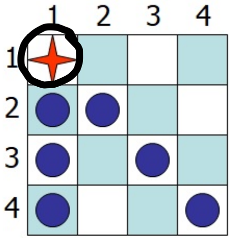
Forward Checking: An extension of backtracking search.
Employs a modest amount of propagation (look ahead).

Intuition: When **instantiating** a variable V, do the following for **all constraints** C that have only **one uninstantiated** variable X remaining:

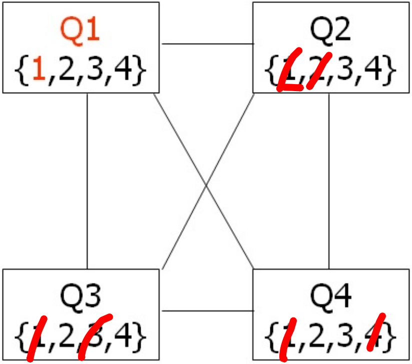
- Check all the values of X;
- Prune those values that violate C.

Undo the pruning when backtrack.

Each of $Q_1, ..., Q_4$ denotes a queen per row.
Forward checking prunes domains of $Q_1, ..., Q_4$ based on binary constraints over $Q_1, ..., Q_4$.

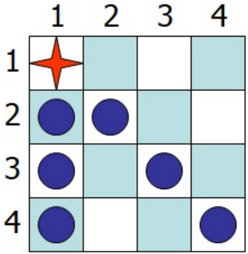


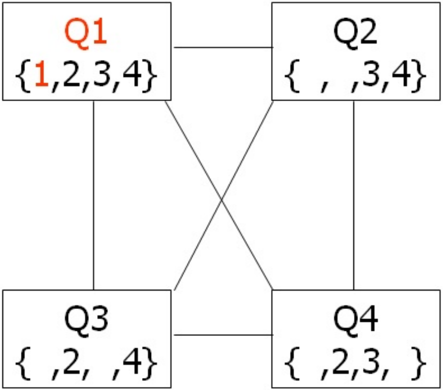
Q_2
Q_3
Q_4

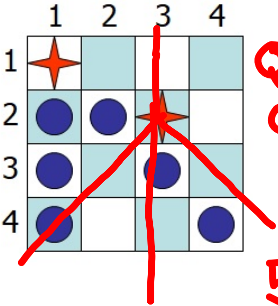


constraints

$c(Q_1,Q_2)$
$c(Q_1,Q_4)$
and so on...


1
2
3
4
1
2
3
4

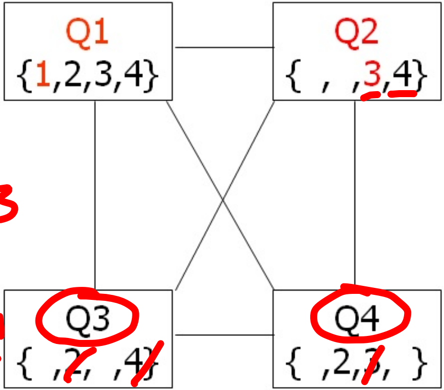

Q1
{1,2,3,4}
Q2
{ , ,3,4}
Q3
{ ,2, ,4}
Q4
{ ,2,3, }



$Q_1 = 1$

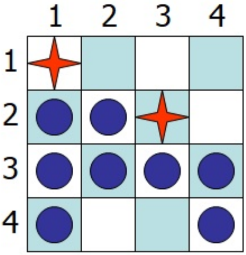
$Q_2 = 3$

DWO!

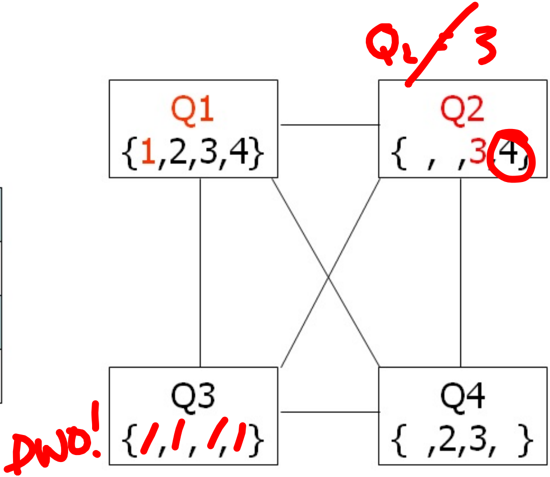


Q_3 post FC?

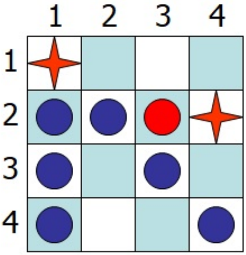
Answer a Question! http://etc.ch/xyX3



1 2 3 4
1
2
3
4

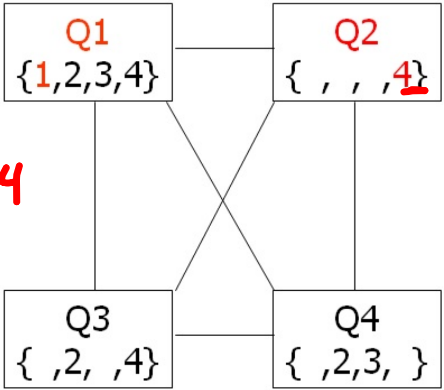


Q1
{1,2,3,4}
Q2
{ , ,3,4}
Q3
{ , , , }
Q4
{ ,2,3, }
DWO!

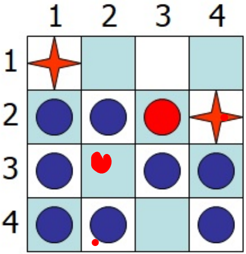


$Q_1 = 1$

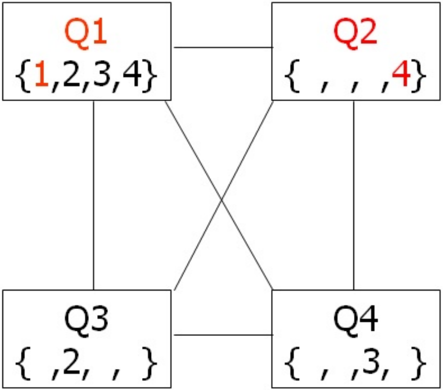
$Q_2 = 4$

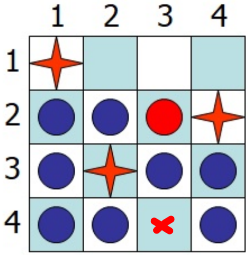


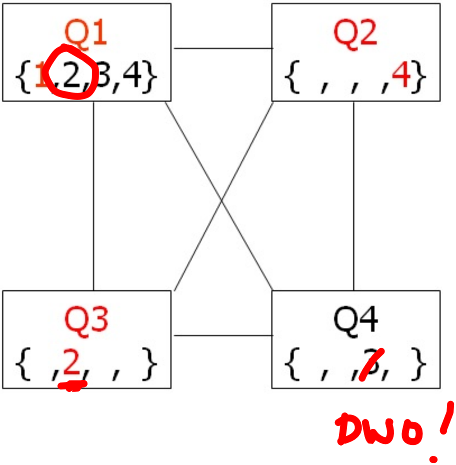
* Remember the pruned vals in case we back track

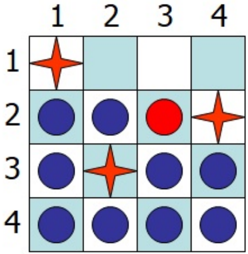


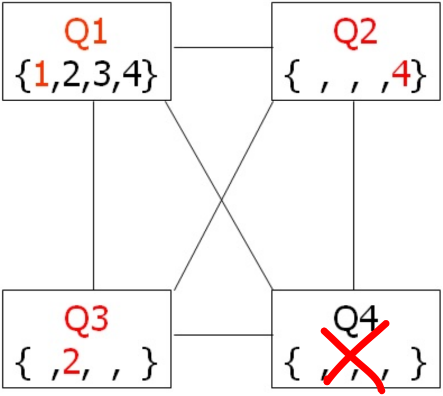
$Q_1 = 1$

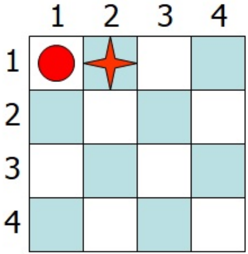


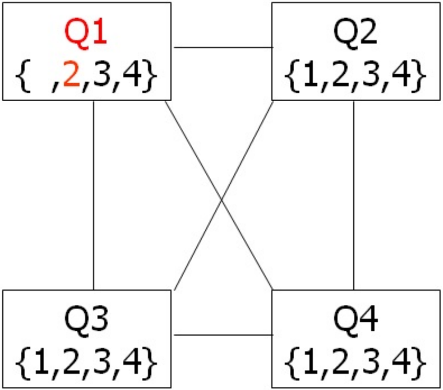

1 2 3 4
1
2
3
4

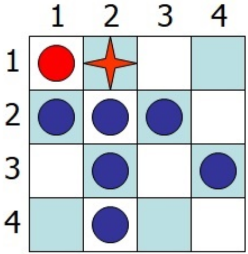

Q1
{1,2,3,4}
Q2
{ , , ,4}
Q3
{ ,2, , }
Q4
{ , ,3, }
DWO!


1
2
3
4
1
2
3
4

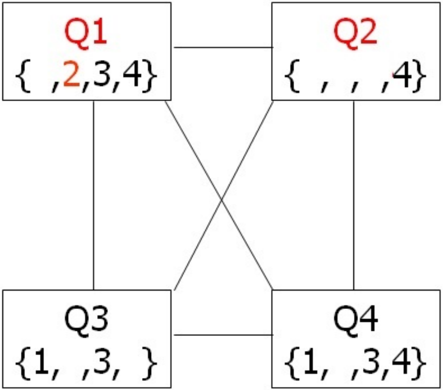

Q1
{1,2,3,4}
Q2
{ , , ,4}
Q3
{ ,2, , }
Q4
{ , , , }


1
2
3
4
1
2
3
4

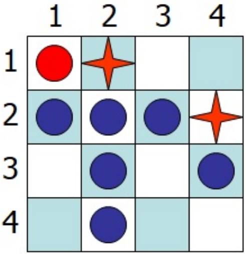

Q1
{ ,2,3,4}
Q2
{1,2,3,4}
Q3
{1,2,3,4}
Q4
{1,2,3,4}



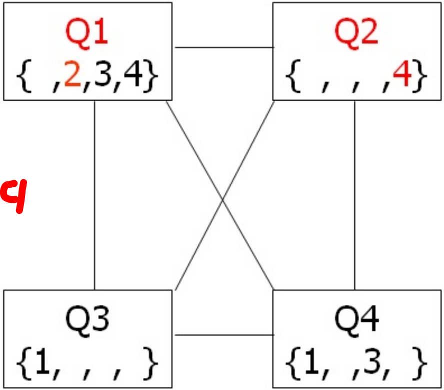
1 2 3 4
1
2
3
4

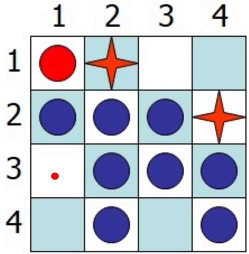


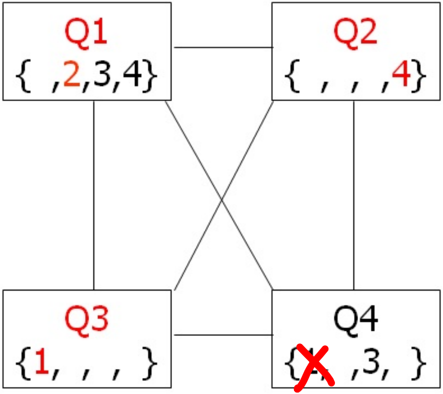
Q1
{ ,2,3,4}
Q2
{ , , ,4}
Q3
{1, ,3, }
Q4
{1, ,3,4}

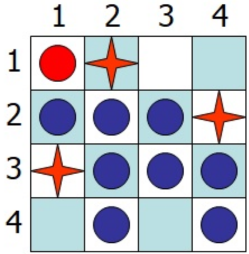


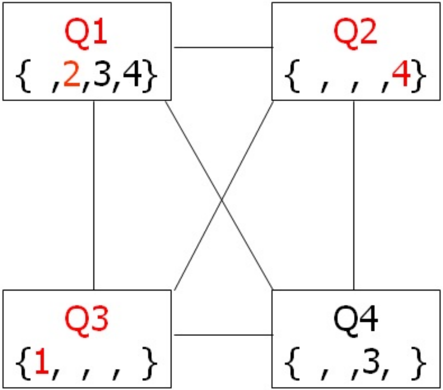
$Q_1 = 2$
$Q_2 : 4$

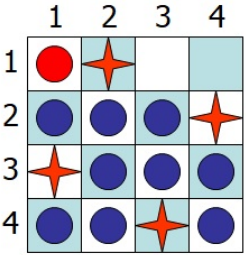



1 2 3 4
1
2
3
4

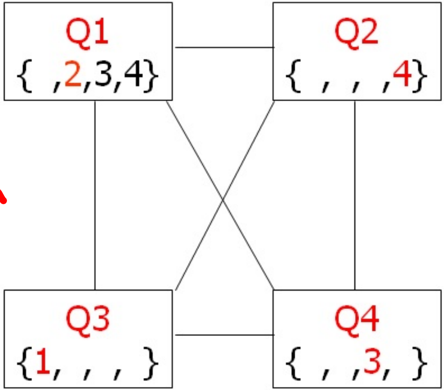

Q1
{ ,2,3,4}
Q2
{ , , ,4}
Q3
{1, , , }
Q4
{ ,3, }


1
2
3
4
1
2
3
4


Q1
{ ,2,3,4}
Q2
{ , , ,4}
Q3
{1, , , }
Q4
{ , ,3, }



Sol'n



Forward Checking: The Algorithm

```
def FCCheck(C,X):
// C is a constraint with all its variables already
// assigned, except for variable X.
1.  for d := each member of CurDom(X):
2.     if making X = d together with previous assignments
                 to variables in the scope of C falsifies C:
3.        remove d from CurDom(X)
4.  if CurDom[X] == {}:
5.     RETURN DWO   # Domain Wipe Out
6.  RETURN ok
```

subroutine for BT-like search

Forward Checking: The Algorithm

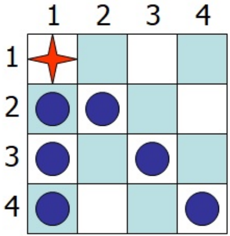
```
def FC(Level):
1.  if all Variables assigned
2.     PRINT Value of each Variable
3.     EXIT or RETURN                        # EXIT for only one solution
                                             # RETURN for more solutions
4.  V := PickUnassignedVariable()
5.  Assigned[V] := TRUE
6.  for d := each member of CurDom(V)
7.     Value[V] := d
8.     DWOoccured:= False
9.     for each constraint C over V such that C has only one
       unassigned variable X in its scope:
10.          if FCCheck(C,X) == DWO:   # X domain becomes empty
11.             DWOoccurred:= True
12.             BREAK                  # stop checking constraints
13.     if NOT DWOoccured:         # all constraints were ok
14.          FC(Level+1)
15.     RestoreAllValuesPrunedByFCCheck()   * book-keeping
16.  Assigned[V] := FALSE     # UNDO as we have tried all of V's values
17.  RETURN
```

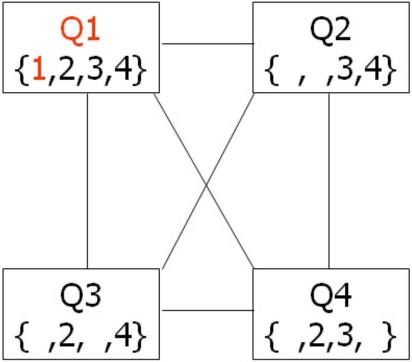
Forward Checking: Restoring Values

- After we **backtrack** from the current assignment the values that were pruned (as a result of that assignment) must be **restored**.

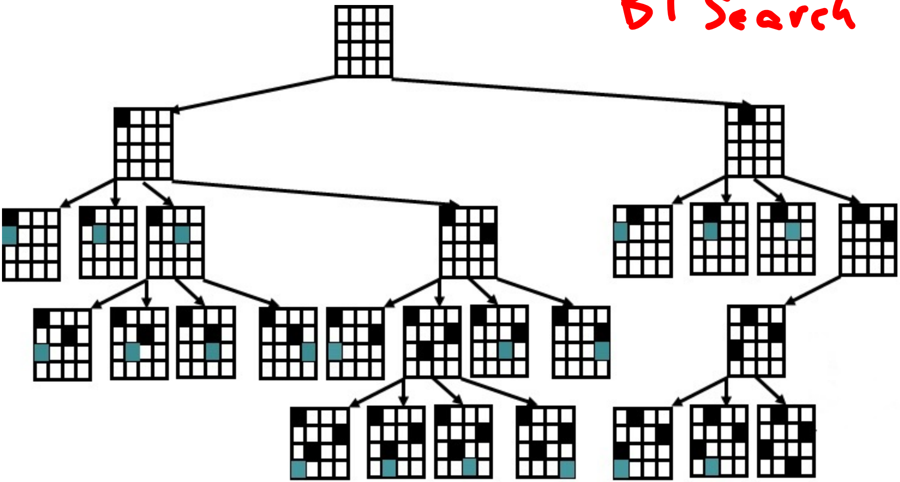
requires... book-keeping!!

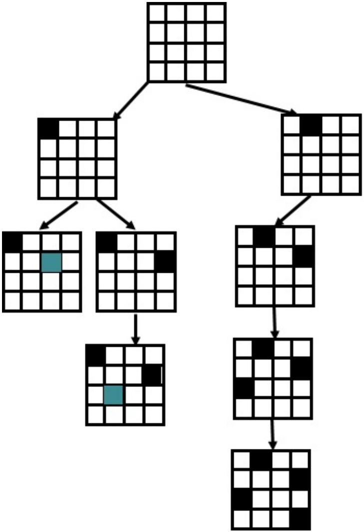
- Some **bookkeeping** needs to be done to remember which values were pruned by which assignment.


1
2
3
4
1
2
3
4


Q1
{1,2,3,4}
Q2
{ , ,3,4}
Q3
{ ,2, ,4}
Q4
{ ,2,3, }

BT Search





FC

effectively reduced

b

Answer a Question! http://etc.ch/xyX3

BT vs FC: Time Efficiency

- The general class of CSPs are **NP-complete**.
  That is, their worst-case running time is **exponential**.
  BT worst-case running time: $\mathcal{O}(d^N)$, where d is the max size of a variable domain, and N is the number of variables.

- But, typically, every NP-complete family contains large sub-classes of simpler problems.

- The purpose of developing constraint propagation techniques, such as FC, is to solve those simpler sub-classes faster.

- FC **often** is about 100 times faster than BT, but it can also do worse!

More on this:
Bacchus, Fahiem, and Adam Grove. "On the forward checking algorithm." International Conference on Principles and Practice of Constraint Programming. Springer, Berlin, Heidelberg, 1995.

What variables would you try first?

	A	B	C	D	E	F	G	H	I
1	8	1	5	6	E1				4
2	6				7	5		8	
3					9				I3
4	9				4	1	7		
5		4						2	
6			6	2	3				8
7					5				
8		5		9	1				6
9	1				E9	7	8	9	5

$\{1, 2, \ldots, 9\}$

$E1 = \{2\}$ *

$E9 = \{2, 6\}$

Answer a Question! http://etc.ch/xyX3

Variable and Value Ordering Heuristics

- Heuristics can be used to determine

  - the **order** in which **variables** are assigned:
    PickUnassignedVariable()

  - the **order** of **values** tried for each variable.

- The choice of the next variable can vary from branch to branch.
  Example: Under the assignment $V_1 = a$ we might choose to assign V_4 next, while under $V_1 = b$ we might choose to assign V_5 next.

- This dynamically chosen variable ordering has a tremendous impact on performance.

Degree Heuristic: Select the variable that is involved in the largest number of constraints on other **unassigned** variables.

Minimum Remaining Values Heuristics (MRV):

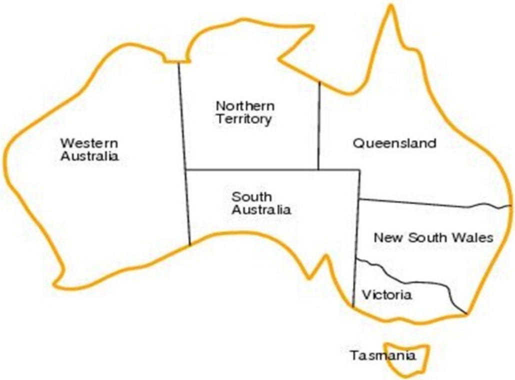
- Always branch on a variable with the smallest remaining values (smallest CurDom).
  Intuition: If a variable has only one value left, that value is forced, so we should propagate its consequences immediately.

- This heuristic tends to produce skinny trees at the top.
  More variables can be instantiated with fewer nodes searched.

- More constraint propagation/DWO failures occur when the tree starts to branch out.
  Hence, inconsistencies can be found much faster.

Example: Map Colouring

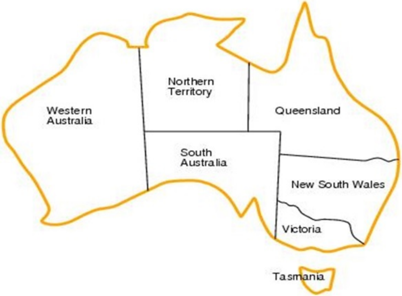
Problem Statement: Color the following map using **red**, **green**, and **blue** such that **adjacent regions have different colors**.

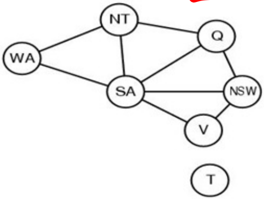


Problem formulation:

- **Variables:** NT, WA, SA .... etc.
- **Domains:** r, g, b
- **Constraints:** WA ≠ NT, WA ≠ SA, Q ≠ SA
  NT ≠ SA, etc.

constraint graph

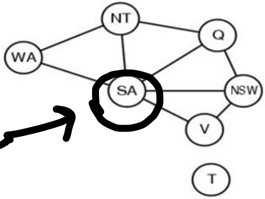




Answer a Question! http://etc.ch/xyX3

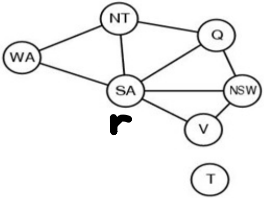
Degree Heuristic

Selects

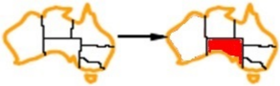


. $\{{\color{red}SA = red}\}$ (using Degree Heuristic)

Answer a Question! http://etc.ch/xyX3



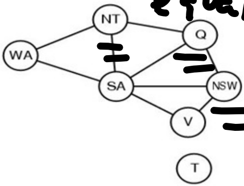
r

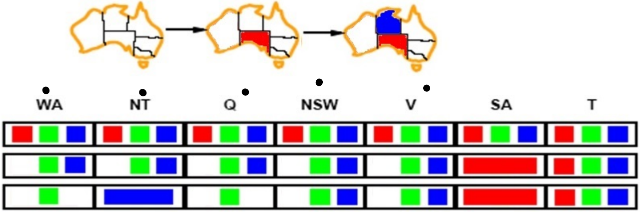


Dom

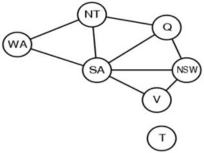
$\{SA = red, NT = blue\}$ (using MRV and Degree Heuristic results in tie between NT, Q and NSW. We choose NT).

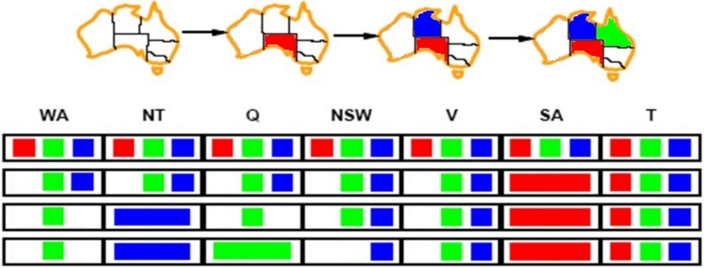
Degrees are equal



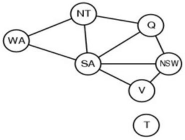


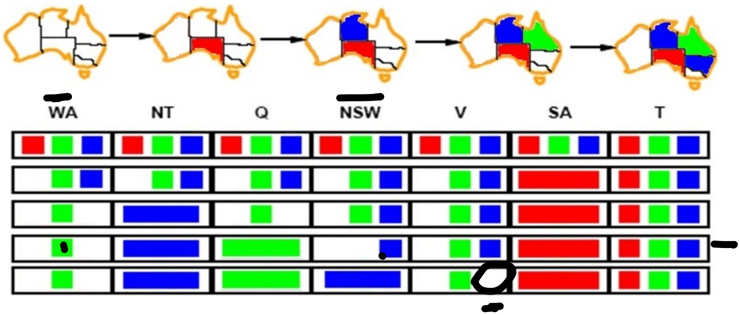
- $\{SA = red, NT = blue, Q = green\}$ (using MRV and Degree Heuristic)





- $\{SA = red, NT = blue, Q = green, NWS = blue\}$ (using MRV and Degree Heuristic)

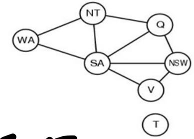




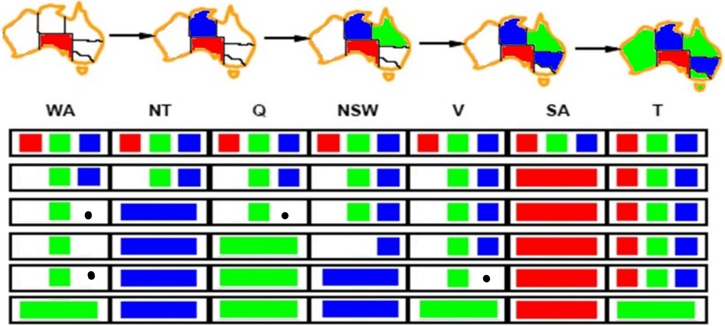
. $\{SA = red, NT = blue, Q = green, NWS = blue, V = green, WA = green, T = green\}$

r,g,b

constraint graph



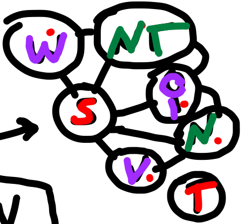
C(NT,WA)

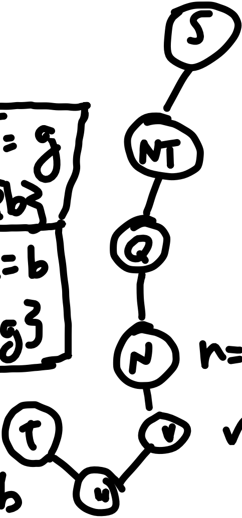


Example: Map Colouring

Variables, Values, Constraints

r, g, b





s = r

W, NT, Q, N, V
{~~g~~ b}

NT = g
W, Q = {b}

Q = b

N = {g}

n = g   V = {b}

v = b

T = r

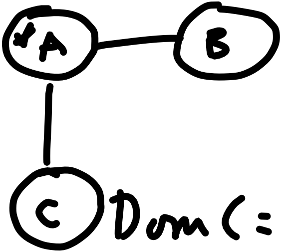
W = b

MRV min. remaining values } Pick Var.

PH

LCV — least constraining value } Pick Val

Dom A: {r, g, b}



Dom B = {g, b}

Dom C = {g, b}

r is the least constraining value

FC and MRV: Empirically

- FC often is about 100 times faster than BT.
- FC with MRV (Minimal Remaining Values) often 10000 times faster.
- On some problems the speed up can be much greater.
  Converts problems that are not solvable to problems that are solvable.
- Still FC is not that powerful.
  Other more powerful forms of constraint propagation are used in practice.