

Calculation of probabilities in a contact-network

Consider the contact network depicted in Figure 21.8(a) as in the text and also on slide 9 of Lecture 43. Let's compute the probability that nodes v, w will contract the disease on the days that they can contract the disease.

Let p be the probability of the disease spreading from some node a to node b on any day that a and b are in contact. We assume that u became infected on day 1.

Let $P(a, t)$ be the probability that node a first becomes infected on day t and hence is contagious for days $t, t + 1, t + 2, t + 3, t + 4$.

For node v , we have to consider $t = 1, 2, 3, 4, 5$. It should be easy to see that $P(v, t) = (1 - p)^{t-1} * p$ for $1 \leq t \leq 5$ and $P(v, t) = 0$ for $t > 5$.

Continuation of calculation

Let $P(w, t)$ similarly denote the probability that w first becomes infected at time t . This will be non-zero precisely for those days t such that v can still be contagious at time t and v and w are in contact at time t . That is, $t = 7, 8, 9$ since v is no longer contagious at time $t = 10$. So

$$P(w, 7) = [P(v, 3) + P(v, 4) + P(v, 5)] * p$$

$$P(w, 8) = [P(v, 4) + P(v, 5)] * (1 - p) * p \text{ and}$$

$$P(w, 9) = [P(v, 5)] * (1 - p)^2 * p$$

For definiteness and simplicity, let $p = 1/2$.

Then $P(v, 1) = 1/2$, $P(v, 2) = 1/4$, $P(v, 3) = 1/8$, $P(v, 4) = 1/16$ and $P(v, 5) = 1/32$ and the probability that v will ever be infected is

$\sum_{1 \leq t \leq 5} P(v, t)$ which is also equal to $1 - (1 - p)^5 = 31/32$.

$P(w, 7) = 7/32 * 1/2 = 7/64$, $P(w, 8) = 3/32 * 1/2 = 3/64$ and

$P(w, 9) = 1/32 * 1/2 = 1/64$ and the probability that w will ever be infected is $11/64$.