Markets and Information

CSC200 Lecture 44

Winter 2015

CSC200: Lecture 44

- This week:
  - Markets and information (Ch.22 excluding Section 22.10)
- Announcements
  - We do not have a simple way to indicate excused absences for work not submitted. But we do maintain a record of all such absences. I will respond to any email requests to verify the absence and plan to bring the list to the final examination.
  - Please do check all grades on the cdf web site. And please maintain all work that has been graded and returned in case there is a discrepancy with the grade being recorded and that which appears on the work.

- Acknowledgement:
  - Some slides' materials are borrowed from the slides of a previous offering of this course. Thanks to Professor Boutilier!

Markets and Information

- So far, we've seen many settings in which individuals' expectations/predictions affect their behavior:
  - Braess' paradox Ch.8: predicting traffic flow, then *chose* a route
  - Information cascades Ch.16: predicting the quality of restaurant to dine at.
  - Network effects Ch.17: predicting how many others will say buy a product (participate in a blog, etc) and then deciding whether or not to buy/join.
  - Commonalities:
    - Individuals need to decide on actions without knowing exactly what will happen/ is happening.
    - Individuals' expectations for payoffs influence how they will choose.
  - Differences:
    - Exogenous: the quality of an alternative/option is intrinsic regardless of what other people do; e.g., in information cascade, the restaurant is good or bad.
    - Endogenous: the desirability of an option depends on the actual decisions others make; e.g., direct benefits, Braes's paradox.

Markets and Information

- We'll examine similar types of phenomenon in *market* settings.
- Can markets aggregate or convey information across a population?
- How do markets aggregate information across populations?
  - Each individual comes with certain beliefs and expectations about the market.
    - About the quality of products or likelihood of events occur.
  - The markets combine this set of beliefs/expectation into an overall outcome (generally into market prices)
- This lecture: How do markets aggregate opinions about exogenous events?

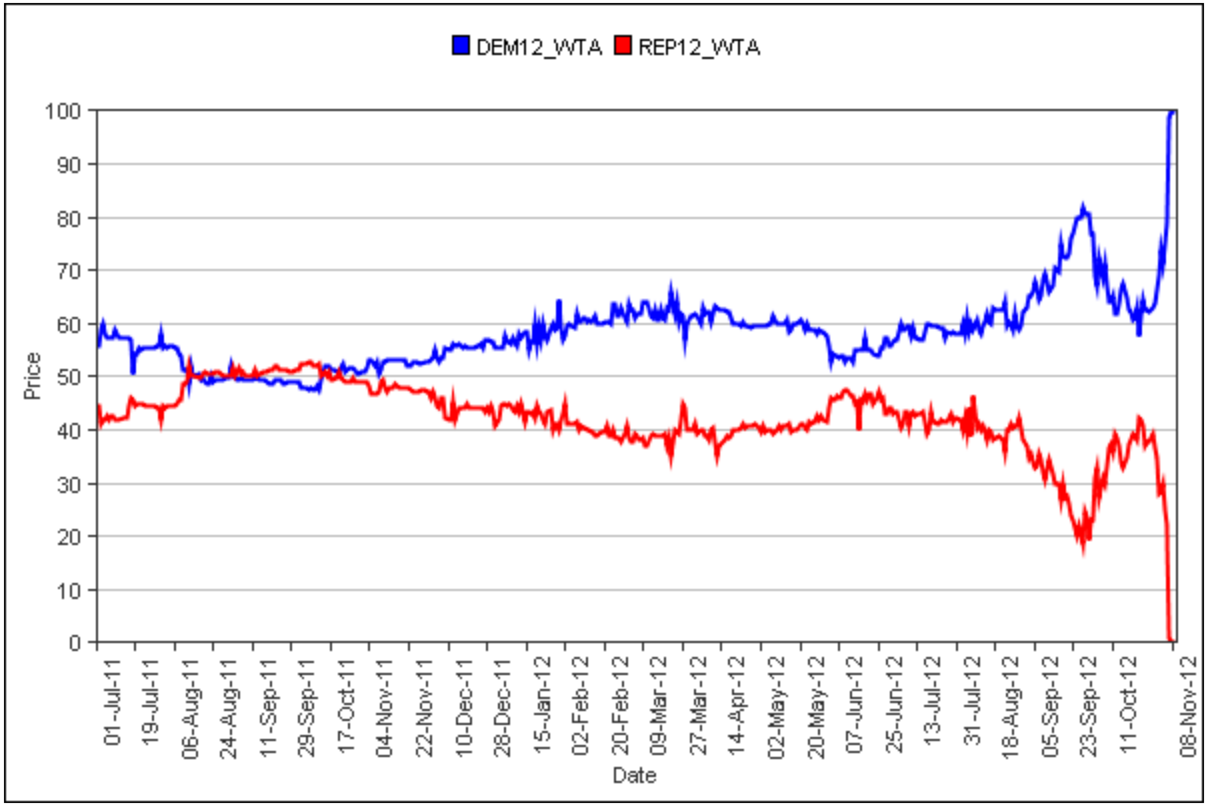
Toy Example: Prediction Market

- Consider a very simple *prediction market*:
  - I offer to sell you a "lottery ticket" : you get \$1 if "Donald Trump wins 2016 US Federal Election"
  - How much should you pay?
    - If you believe Trump wins with probability p, should pay up to $\$p$ (ignoring *risk attitude*)
    - Expected payout is $1*p + 0*(1-p) = \$p$ (e.g., \$0.75 if $p = .75$)
  - If you own the ticket/contract, you should be willing to sell to anyone willing to pay at least $\$p$:
    - If they buy for $\$q > \$p$, they must believe in Trump winning more than you
  - You decide later to sell for less than $\$p$: your beliefs must have changed.

Prediction Markets

- A *prediction market*:
  - Markets for assets/contracts which have been created to aggregate individuals' beliefs/prediction about a future event.
    - predicting presidential elections (Iowa Electronic Market).
  - Participants trade such contracts based on future event beliefs.
  - Two sides to a trade: seller and buyer.
  - A price of a trade captures the belief of traders
    - E.g., if you sell your "Trump wins" contract with price of 0.8$ to me but don't sell it with 0.7$, what can be inferred about our beliefs?
    - "Price separates the belief"
- Goal: the eventual market prices converge to the "collective belief" of the population for the future event.
  - If price of "Trump wins" contract is 60 cents, this means that the market believes that the probability for Trump winning is 0.6.

Prediction Markets in practice



2012 US Presidential Election Market
From *Iowa Electronic Markets*
http://tippie.uiowa.edu/iem/markets/data_pres12.html

Other markets and information

- In other markets besides prediction markets, information is aggregated as market price.
  - Betting markets: individuals bet on sport events such as horse races, etc.
  - Stock markets in which stocks of various companies are sold at various prices.
    - Stock price captures what the population believes about the future of a company. Higher price in stocks of a company usually are indication of (future) prediction of success for that business.
- We start by formalizing a simple but stylized model of a betting market on horse race.

Two Forms of Odds ("Odds Against")

- **E&K's odds:** multiplicative factor payout on bet of \$d
  - e.g., 3:2 odds means 3/2 = 1.5 rate of *total* return on bet
  - e.g., give the track *\$d* and get *\$(1.5*d)* in return if horse wins
  - e.g., give the track 10\$, what is the return if horse wins?
  - Note: these odds can never be less than 1 given this definition

- **Bookmaker odds**: payout of a bet *excluding* stake
  - e.g., 3:2 odds means a \$2 bet will win you \$3 plus original stake
  - e.g., at 3:2 odds, you give track \$2 and return is \$5 if you win
  - Note: these odds can be less than 1; e.g., at 1:3 odds a \$3 bet wins \$1 (plus return of original stake); in fact, *EK odds = 1+BM odds*

Horse Racing Example

- A two-horse race with horses *A* and *B*: One wins the race (no ties).
- Bettor *G* has *$w* to bet on these horses, how to bet?
- G bets $r \in [0,1]$ fraction of $w on A and $1 - r$ on B, how? Depends on:
  - *Odds* given on each horse by the track: O_A and O_B
  - *G*'s *beliefs* about each horse winning: $Pr(A) = a$, $Pr(B) = b = 1-a$

Odds Given	O_A	O_B
Bettor Beliefs (Pr of Winning)	a	$b = 1-a$
Fraction of wealth w placed on horse	r	$1-r$
Payout if Horse Wins	$O_A wr$	$O_B w(1-r)$

Utility and Risk Attitudes

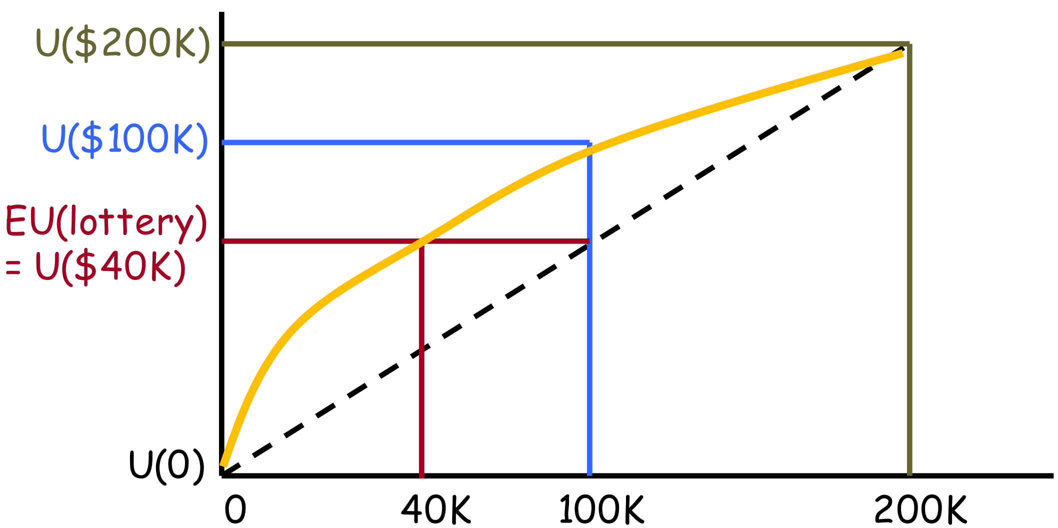
- Assume the bettor evaluates a bet according to the expected value of the payoff on the bet.
- First before any analyses, we should decide on how people value money, or what is their payoff of a given wealth w? Recall St. Petersburg paradox
  - Define utility fn. U(.): mapping the wealth of a bettor to his payoff.
  - Which utility function do you suggest?
- The simplest utility functions are linear functions such as $U(w)= w$ or $u(w) = aw + b$:
  - Not aligned with empirical evidence.
  - Not aligned with common sense.

Utility and Risk Attitudes

- A *fair bet* is one where expected payout of a bet is the bet itself.
- Would you always take a fair bet?
- Would you pay \$1 for a 50% chance of winning \$2?
  - A: \$1 or B: [(.5, \$2), (.5, 0)]
- What would you choose?
  - A: \$100,000 or B: [(.5, \$200,000), (.5, 0)]
  - what if B was \$250K, \$300K, \$400K, \$1M ?
  - what if p = .6, .7, .9, .999, …
- generally, *utility* *U(**EMV**(gamble)) > U(gamble)*
  - *EMV = expected monetary value*
- Utility of money is nonlinear: e.g., *U(\$100K) > 0.5U(\$200K) + 0.5U(\$0)*
  - *That is why U(w) = aw+b is not of interest.*

Utility and Risk Attitudes

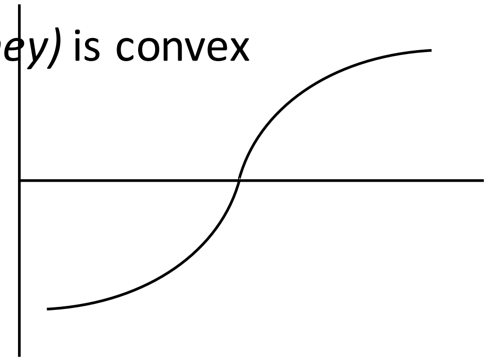
- What would you choose?
  - A: \$100,000 or B: [(.5, \$200,000), (.5, 0)]
- Yellow curve represents *utility* for a specific amount of money
- Note: Utility of a certain payout of \$100K is higher than the 50-50 average of the U(0) and U(200K) (mid-point of dotted line)
- *Certainty equivalent* of gamble *g*: $U(CE) = U(g); CE = U^{-1}(EU(g))$



For many people, CE ~ \$40K
Note: 2nd \$100K "worth less" than 1st \$100K

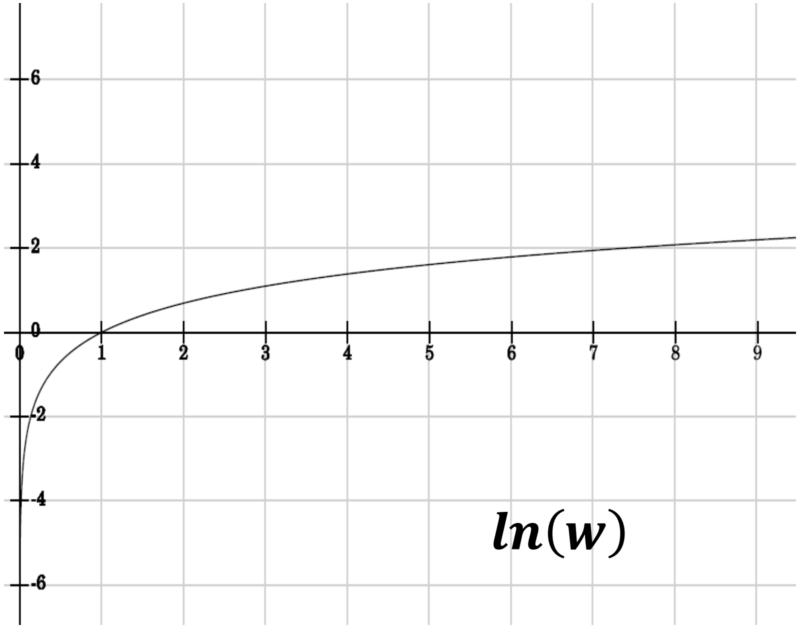
Risk attitudes

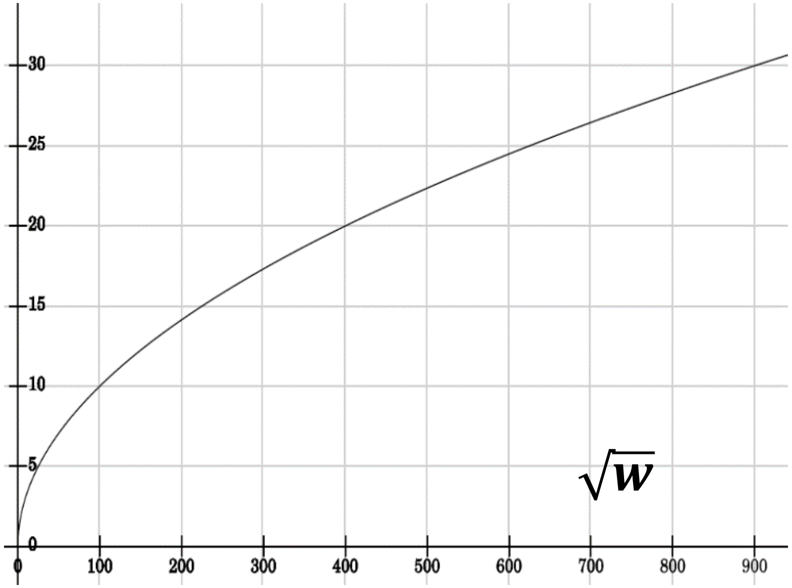
- *Risk Premium: EMV(g) – CE(g)*
  - how much of EMV of gamble *g* will I give up to remove risk of losing
  - E.g., g = [(0.5, 2M$), (0.5,0$)], CE(g) = 600K$, what is risk premium?
- Risk averse:
  - decision maker has positive risk premium; *U(money)* is concave
- Risk neutral:
  - decision maker has zero risk premium; *U(money)* is linear
- Risk seeking:
  - decision maker has negative risk premium; *U(money)* is convex
- Most people are risk averse (in positive range)
  - this explains insurance
  - often risk seeking in negative range



Possible Risk Averse Utility Functions

- For non-negative payouts w, $\sqrt{w}$ or $\ln(w)$ are both concave and represent risk aversion (utility grows at a decreasing rate as a function of w)
- Logarithmic utility, *U(w) = ln(w),* has a nice property:
  - note: $ln(w_1) - ln(w_2) = ln(w_1/w_2)$
  - So utility of increased wealth by constant factor m is independent of w
    - *ln(mw) - ln(w) = ln((mw)/w) = ln(m)*
    - *e.g., doubling wealth has same increase in utility, ln(2), regardless of starting pt*
  - Bettor with ln(w) utility function (or any other concave fn) rejects fair bet.





Forced Bets: What is Optimal?

- Assume bettor *G* forced to bet wealth amount w
- Assume logarithmic utility (for illustration purposes)
  - ***What is optimal bet: what fraction r on horse A, 1-r on horse B?***
- First, expected utility of betting this fraction, *r* and *1-r*, is:

$$a\ln(O_A wr) - (1-a)\ln(O_B w(1-r))$$

$$= a\ln(r) - (1-a)\ln(1-r) + a\ln(O_A w) - (1-a)\ln(O_B w)$$

A  B 

Odds Given	O_A	O_B
Bettor Beliefs (Pr of Winning)	*a*	*b = 1-a*
Fraction of wealth w placed on horse	*r*	*1-r*
Payout if Horse Wins	$O_A wr$	$O_B w(1-r)$
Utility if Horse Wins	$ln(O_A wr)$	$ln(O_B w(1-r))$

Forced Bets: What is Optimal?

- Maximizing expected utility by proper choice of *r* : ignore red box (why?)

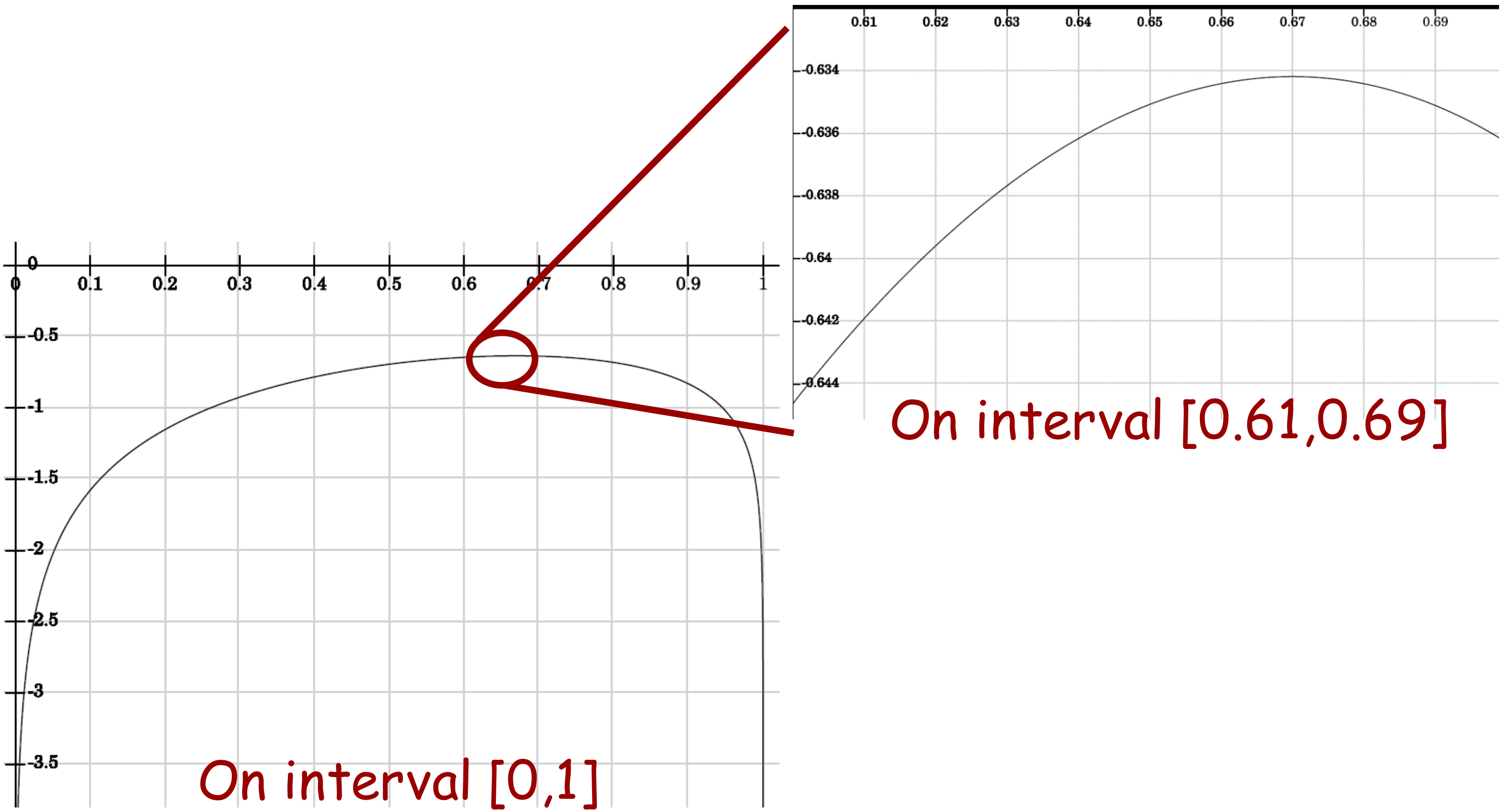
$$a \ln(r) - (1-a)\ln(1-r) + a\ln(O_A w) - (1-a)\ln(O_B w)$$

- The term $a \ln(r) - (1-a)\ln(1-r)$ is maximized at point *r = a (why?)*
- So the optimal bet for *G* is to "bet her beliefs"
  - Place fraction *a* of wealth on A and fraction *b = 1-a* of wealth on B
  - Notice: no dependence on odds (odds are a fixed constant factor increase in wealth due to log utility)

A  B 

Odds Given	O_A	O_B
Bettor Beliefs (Pr of Winning)	*a*	*b = 1-a*
Fraction of wealth w placed on horse	*r*	*1-r*
Payout if Horse Wins	$O_A wr$	$O_B w(1-r)$
Utility if Horse Wins	$\ln(O_A wr)$	$\ln(O_B w(1-r))$

Plot of: $a\ln(r) - (1-a)\ln(1-r)$ for a=0.67



Let's Turn This Into a Market (1)

- Assume n "independent" bettors, 1, …, n
  - Bettor k has wealth w_k, beliefs a_k and $b_k = 1 - a_k$
  - Let total wealth $w = \Sigma_k w_k$; then k has wealth share (fraction) $f_k = w_k / w$
  - Assume all bet their beliefs (e.g., all have log utility and bet optimally)
- Total bets on A: $a_1 w_1 + a_2 w_2 + \ldots + a_n w_n$
- Payout by track if A wins: $O_A(a_1 w_1 + a_2 w_2 + \ldots + a_n w_n)$
- Assume "fair track": pays out exactly all bets w whichever horse wins
- Needs to set the *equilibrium odds* O_A to solve:

$$O_A(a_1 w_1 + a_2 w_2 + \ldots + a_n w_n) = w$$

Inverse odds

$$\equiv \frac{1}{O_A} = \frac{(a_1 w_1 + a_2 w_2 + \ldots + a_n w_n)}{w}$$

$$\equiv \frac{1}{O_A} = a_1 f_1 + a_2 f_2 + \ldots + a_n f_n$$

Let's Turn This Into a Market (2)

- Thus the *inverse odds* on A at a fair track are the *wealth-weighted average beliefs* of the bettors that A will win (similarly for B)
  - The wealthier you are, your belief is more powerful/influential

$$\frac{1}{O_A} = a_1 f_1 + a_2 f_2 + \ldots + f_n w_n$$

$$\frac{1}{O_B} = b_1 f_1 + b_2 f_2 + \ldots + b_n f_n$$

- Inverse odds called *state prices*: $\rho_A = {}^{1}/_{O_A}$
  - "price of 1 dollar": price one must pay to earn \$1 if A wins
- To ensure return of \$1: a bettor *k* must bet ρ_A on A and bet ρ_B on B

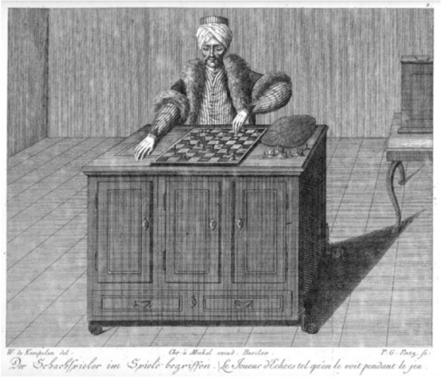
$$\begin{aligned}\frac{1}{O_A} + \frac{1}{O_B} &= (a_1 f_1 + a_2 f_2 + \ldots + a_n f_n) + (b_1 f_1 + b_2 f_2 + \ldots + b_n f_n) \\ &= (a_1 + b_1) f_1 + (a_2 + b_2) f_2 + \ldots + (a_n + b_n) f_n \\ &= f_1 + f_2 + \ldots + f_n \\ &= 1\end{aligned}$$

- State prices sum to 1; and bettor forced to bet can always get his money back with certainty!

Betting Doesn't Work This Way…

- Bookies/tracks do not set odds to be fair
- Bets are not independent
  - Bets are placed at different times, semi-sequentially
  - Odds at track change as bets are placed…
  - Bettors aren't always risk averse (nor log utilities if risk averse)…
  - Later bettors learn from bets of earlier bettors and should update their beliefs
  - The process by which beliefs are updated is not easy to formalize… do bettors have same or different information? Are prior bets correlated?
  - What about decision "when to place bet"?
- In general prediction markets, some bettors can manipulate the market (especially if they have strong information): place bets to change odds in a particular way (possibly to influence the beliefs of others… remember information cascades?) and then exploit the odds

Crowdsourcing/wisdom of crowds



- Recent trend toward *online crowdsourcing*:
- Use large group of people, each often working on "microtasks", to solve large problems
  - e.g., translating a document, labeling images, ...
  - Amazon's Mechanical Turk, CrowdFlower, ...
- How to verify quality of work?
  - use the crowd to place bets of quality of result
  - "Wisdom of the Crowds" (see James Surowiecki's 2004 book)

Take Home Message

- A prediction market that provides payouts conditional on some future event can be interpreted as aggregating predictions/beliefs of an entire population if they act to maximize their expected utility.
- State prices represent “market beliefs”.
- Cautions/observations:
  - If all bettors have same beliefs, this will be market belief
  - If “accuracy” of beliefs correlated with wealth share, markets beliefs may be skewed