Social Choice

CSC200 Lecture 40

March 21, 2016

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Announcements

- Announcements

I have made several clarifications regarding Assignment as I will discuss in class.

It is still due March 30

On Wed, 11 AM, Nisarg Shah will be giving a seminar in BA 1220. The title is “Optimal Social Decision Making”

He is an expert in social choice/voting so if you have time you should consider attending.

Todays agenda

- The axiomatic approach and impossibility results: Arrows Theorem, Muller-Satterthwaite
- Manipulation of elections
- Gibbard-Sattherwaite impossibility theorem
- Can we restrict preferences further to get good voting rules?
- Single-peaked preferences

  - Readings: Ch.23 (plus important ideas not discussed in the text)

Independence of Irrelevant Alternatives (IIA)

- Another view of IIA: suppose *a* wins over *b* in an election. Then we add a new alternative. *Without changing anyone's relative preferences for a and b*, suddenly *b* can win.

- Consider the following preferences:
  - 100 votes: Bush $\succ$ Gore $\succ$ Nader
  - 12 votes: Nader $\succ$ Gore $\succ$ Bush
  - 95 votes: Gore $\succ$ Nader $\succ$ Bush
  - Run a plurality election with only two candidates, Bush and Gore
    - Gore wins over Bush (plurality score of 107 to 100)
  - At the least minute, Nader enters the race:
    - Bush wins the election now (plurality score of 100 to 95 to 12)

Other Principles

- *Unanimity:* if all $v \in V$ rank a first, then a wins
  - relatively uncontroversial (unfortunately sometimes called weak Pareto)
- *Weak Pareto:* if all $v \in V$ rank $a \succ b$, then b cannot win
  - relatively uncontroversial
- *Non-dictatorial:* there is no voter k s.t. a is the winner whenever k ranks a first (no matter what other voters say)
- *Anonymity:* permuting votes within a profile doesn't change outcome
  - e.g., if all votes are identical, but provided by "different" voters, result does not change (can't depend on voter's identities)
  - implies non-dictatorship
- *Neutrality:* permuting alternatives in a profile doesn't change outcome
  - i.e., result depends on relative position of an alternative in the votes themselves, not on the identity of the alternative
  - implies non-imposition

Arrow's Theorem

- So can we satisfy all (or even some of these axioms)?
- **Arrow's Theorem (1951):** Assume at least three alternatives. No voting rule can satisfy IIA, weak Pareto, and non-dictatorship.
  - Most celebrated theorem in social choice
  - Broadly (perhaps too broadly) interpreted as stating there is no good way to aggregate preferences
  - Key point: Arrow's Theorem is phrased in terms of a rule producing a ranking. We will soon see impossibility results for when we just want a single winner.
- There are a wide variety of alternative proofs around
  - Easley and Kleinberg provide one proof (see Ch. 23.11)
  - I include an especially simple proof in the next two slides for anyone interested in digging into the details

Brief Proof Sketch (Optional)

- Fix SWF F; let $\succ_F$ denote social preference order given input profile
- A coalition $S \subseteq N$ is *decisive* for a over b if, whenever $a \succ_k b, \forall k \in S$, and $a \nsucc_j b, \forall j \notin S$, we have $a \succ_F b$.
- **Lemma 1:** if S is decisive for a over b then, for any c, S is decisive for a over c and c over b.
- **Sketch:** Let S be decisive for a over b.
  - Suppose $a \succ_k b \succ_k c, \forall k \in S$ and $b \succ_j c \succ_j a, \forall j \notin S$.
  - Clearly, $a \succ_F b$ by decisiveness.
  - Since $b \succ_j c$ for all j, $b \succ_F c$ (by unanimity), so $a \succ_F c$.
  - If b placed anywhere in ordering of *any agent,* by
  - IIA, we must still have $a \succ_F c$.
  - Hence S is decisive for a over c.
  - Similar argument applies to show S is decisive for c over b.

$S: b \succ a \succ c$
$-S: c \succ a \succ b$ $\Rightarrow$ $F: a \succ c$

- **Lemma 2:** If S is decisive for a over b, then it's decisive for every pair of alternatives $(c,d) \in A^2$
- **Sketch:** By Lemma 1, S decides c over b. Reapplying Lemma 1, S decides c over d.

Brief Proof Sketch (Optional)

- So now we know a coalition *S* is either *decisive* for all pairs or for no pairs.
- Notice that *entire group N is decisive for any pair of outcomes* (by unanimity)
- **Lemma 3:** For any $S \subseteq N$, and any partition (T,U) of S. If S is decisive then either or T is decisive or U is decisive.
- **Sketch:** Let $a \succ_k b \succ_k c$ for $k \in T$; $b \succ_j c \succ_j a$ for $j \in U$; $c \succ_q a \succ_q b$ for $q \in N \backslash S$;
  - Social ranking has $b \succ_F c$ since S is decisive.
  - Suppose social ranking has $a \succ_F b$, which implies $a \succ_F c$ (by transitivity).
    - Notice only agents in T rank $a \succ c$, and those in U, $N \backslash S$ rank $c \succ a$.
    - But if we reorder prefs for any other alternatives (keeping $a \succ c$ in T, $c \succ a$ in U *and* $N \backslash S$), by IIA, we must still have $a \succ_F c$ in this new profile.
    - Hence T is decisive for a over c (hence decisive for all pairs).
  - Suppose social ranking has $b \succ_F a$
    - Since only agents in U rank $b \succ a$, similar argument shows U is decisive.
  - So either T is decisive or U is decisive.

End of proof

- **Proof of Theorem:** Entire group N is decisive. Repeatedly partition, choosing the decisive subgroup at each stage. Eventually we reach a singleton set that is decisive for all pairs… the dictator!

Muller-Satterthwaite Theorem

- Arrow's theorem: impossible to produce a societal ranking satisfying our desired conditions
  - What if we only want a unique winner?
  - Also not possible…

- **Muller-Satterthwaite Theorem (1977):** Assume at least three alternatives. No resolute (one that doesn't produce ties) voting rule can satisfy strong monotonicity, non-imposition (unanimity), and non-dictatorship.

May’s Theorem

- Complete despair? Not really. We could either:
  - dismiss some of the axioms/properties as too stringent
  - live with “general” impossibility, but use rules that tend to (in practice) give desirable results (behavioral social choice)
  - look at restrictions on the assumptions (number of alternatives, all possible preference/vote profiles, …)

- Here’s a positive result (and characterization)…

- **May’s Theorem (1952):** Assume *two* alternatives. Plurality is the only voting rule that satisfies anonymity, neutrality, and positive responsiveness (a slight variant of weak monotonicity).

Manipulation of Elections

- Recall our discussion of mechanism design (e.g., auctions)
  - we needed special mechanisms (e.g., VCG mechanism, 2^{nd}-price auction) to ensure that people would report their valuations truthfully
  - these mechanisms relied on carefully crafted payments
  - in other settings (e.g., 1^{st}-price auction), true valuations are not declared
- In voting (social choice) we don't usually consider payments such as
  - if we go to your restaurant, you need to pick up the bar tab; or if your candidate wins an election, we increase your property taxes 0.3%
  - aside: it's worth noting that VCG was motivated in some circles as a means for taxing for public projects (the "Clarke tax")
- So is it possible for a voter to get a better outcome by misreporting their preferences?

Examples of Manipulability

- As with mechanism design, most voting rules provide positive incentive to misreport preferences to get a more desirable outcome
  - political phenomena such as *vote splitting* are just one example
- Plurality:
  - 100 votes: Bush $\succ$ Gore $\succ$ Nader
  - 12 votes: Nader $\succ$ Gore $\succ$ Bush
  - 95 votes: Gore $\succ$ Nader $\succ$ Bush
  - Bush wins truthful plurality vote; Nader supporters are better off voting for Gore! *Notice that Borda, STV would give election to Gore*
- Borda: same example with different numbers
  - 100 votes: Bush $\succ$ Nader $\succ$ Gore
  - 17 votes: Nader $\succ$ Gore $\succ$ Bush
  - 90 votes: Gore $\succ$ Nader $\succ$ Bush
  - Bush wins truthful Borda vote (B:200 pts; G:197pts); Nader supporters better off ranking Gore higher than Nader! Bush supporters were better off ranking Gore last.

Gibbard-Satterthwaite Theorem

- *Strategyproofness* defined for voting rules just as for mechanisms
  - Informally, there no preference profiles where an insincere report by any voter k (i.e., reporting something other than his true ranking) leads to outcome that is preferred by k to the result obtained from his true report
- Manipulability unavoidable in general (for general SCFs)

- **Thm (Gibbard73, Sattherwaite75):** Let r be a voting rule (over voters N, alternatives A) s.t.:
  - (i) $|A| > 2$;
  - (ii) r is *onto* (every outcome is selected for some vote profile V);
  - (iii) r is non-dictatorial;
  - (iv) all preference profiles (combinations of rankings) are possible.

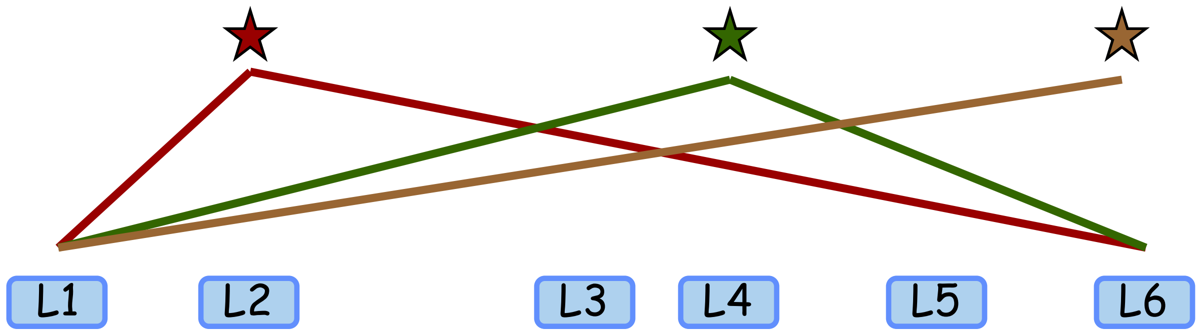
  Then r cannot be strategy-proof.

Are we doomed to possible manipulation?

- Unlike the prevous impossibility theorems, the axioms in the Gibbard Satherthwaite Theorem seem very reasonable.
- But the theorem does imply that all preference profiles are possible which in many applications is not the case.
- Moreover, one of the insights of algorithmic social choice is that while certain voting rules can be manipulated, it may be computationally hard to determine how this manipulation can be done.

Single-peaked Preferences

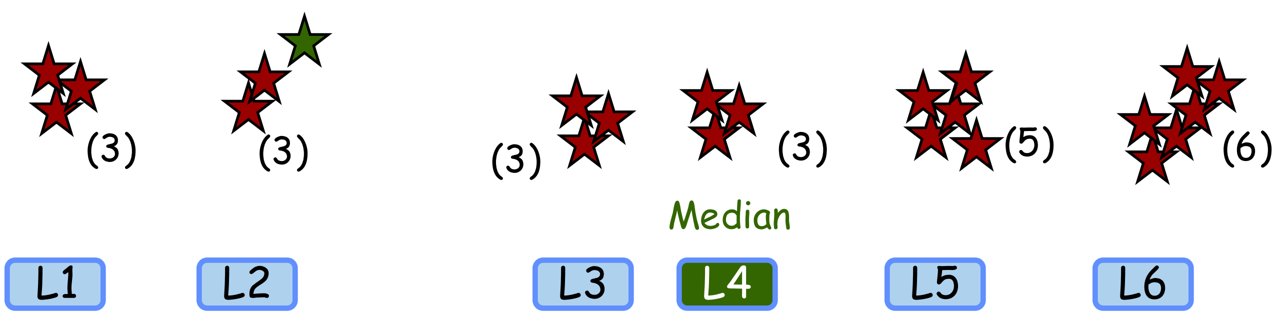
- Special class of preferences for which GS Theorem is circumvented
- Let $\gg$ denote some "natural" ordering over alternatives *A*
  - e.g., order political candidates on left-right spectrum
  - e.g., locations of park, warehouse on a line (e.g., position on a highway)



- Voter *k*'s preferences are *single-peaked* if there an *ideal* alternative, *a*[k],* that *k* likes best, and that as you move away from *a*[k]* in the ordering $\gg$, alternatives become less and less preferred by *k;* that is:
  - $a^*[k] \succ_k a$ for any $a \neq a^*[k]$
  - $b \succ_k c$ if either: (1) $c \gg b \gg a^*[k]$; or (2) $a^*[k] \gg b \gg c$
- In figure: green voter (★) prefers $L4 \succ L3 \succ L2 \succ L1$ and $L4 \succ L5 \succ L6$

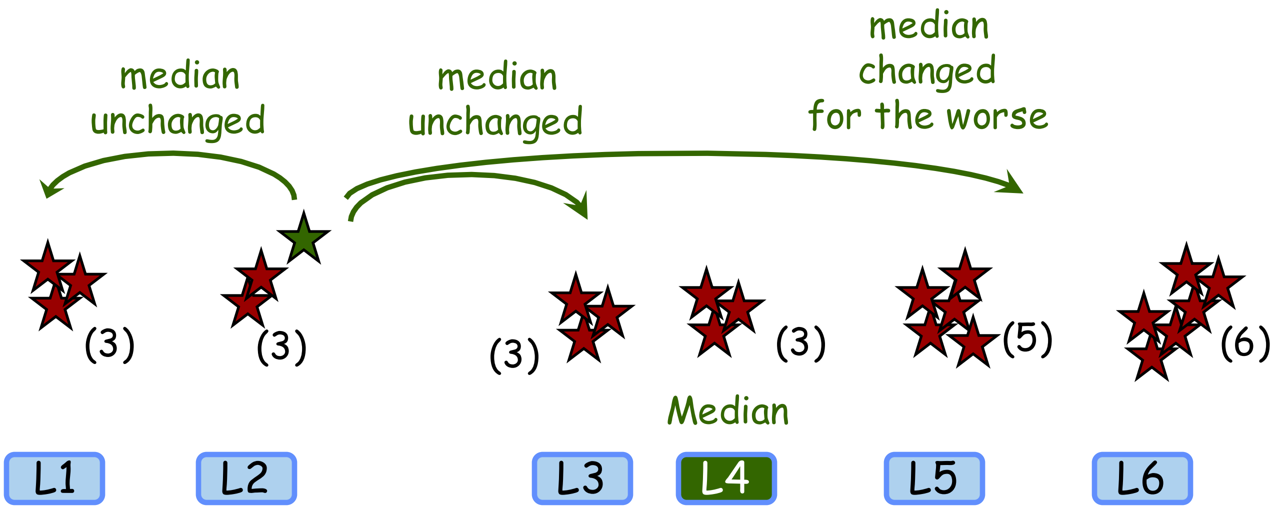
Median Voting

- Suppose all voter's prefs are single-peaked
  - they must be single-peaked w.r.t. the same domain ordering ≫
  - but you can use any ordering you want (as long as it creates SP'ed prefs)
- *Median voting scheme:* voter specifies only her peak; winner is *median* of the reported peaks (Black 1948)



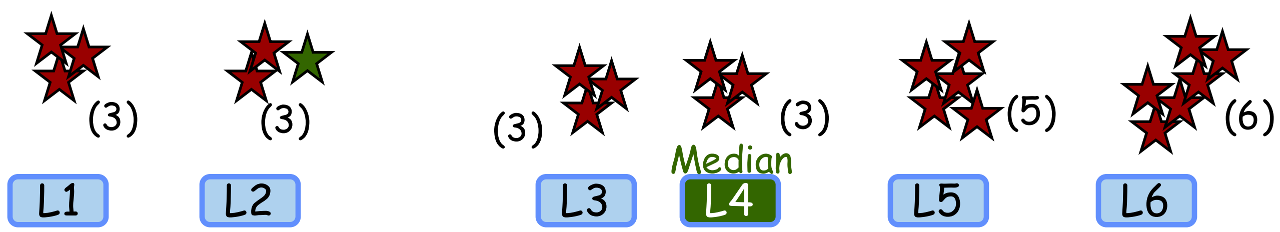
What's Special about Median Voting?

- Assume all voters have single-peaked preferences and we use median voting to determine the winner
- One property: voters don't report full rankings, just peaks (or favorite)
- Another critical property: the voting scheme is *strategyproof*
  - easy to see, let's look at example
  - intuition: if you "lie about your peak" you either report something:
    - on the same side of median as your peak: median unchanged
    - …or on opposite side of median as peak: median moves further away



What's Special about Median Voting?

- Assume single-peaked preferences and use median voting
- The winner *W* is Pareto efficient *(in example L4)*
  - *no other choice is better for one person without hurting someone else*
- The winner *W* is a Condorcet winner (if *n* odd): Why?
  - at least *(n+1)/2* voters prefer *W* to anyone *left* of *W* (more if there is more than one voter's peak at the median)
  - at least *(n+1)/2* voters prefer *W* to anyone *right* of *W* (more if there is more than one voter's peak at the median)
  - ***so W wins a majority election against any other candidate***
- Known as the *Median Voter Theorem*



What's Special about Median Voting?

- Can take Median Voter Theorem a step further, imagine following procedure:
  - place *W* at top of societal ranking, then remove it from candidate set
  - repeat process to find median winner among *remaining* candidates
    - there again must be a Condorcet winner (!)
    - *in example: peaks for all voters stays the same except for those who voted for L4 (those voters each have a new peak, either L3 or L5)*
  - remove and repeat until you've ranked all candidates
- Societal ranking must be complete and transitive and respects majoritarian preferences: if A >B in ranking, the majority prefer A to B
  - breaks the Condorcet paradox