Social Choice

CSC200 Lecture 39

March 16, 2016

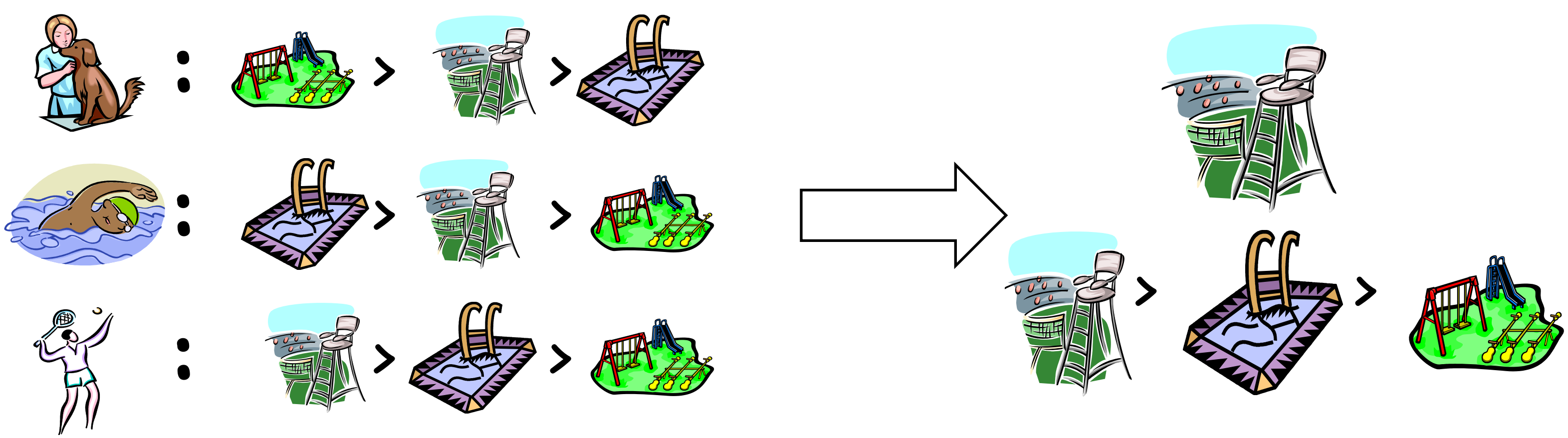
Allan Borodin

Todays agenda and announcements

- Today: Review of social choice and voting. More voting rules, Axioms, Manipulation, Single-peaked preferences
  - Reading for next two weeks: Ch.23 (plus some important ideas not discussed in the text)
  - This week: 23.1-23.6; next week: 23.7-23.10
- Announcements
  - Final quiz (quiz 8) scheduled for April 1. Aids allowed for the final exam is the same as for all quizzes and tests; one 8.5 by 11 sheet (2 sides) of *handwritten* notes are the only aids allowed.
  - As in all assignments, quizzes and tests, you will receive 20% credit for any question (or question part) where you explicitly state “I do not know how to answer this question”.
  - Last assignment is due March 30 and has been posted.
  - Office hours by appointment for next couple weeks.

Voting and Preference Aggregation

- Last time
  - Introduced *social choice*: preference aggregation to make a single "consensus" decision for a group
  - The concept of a *voting rule*:
    - Given: a set N of n voters and a set A of m alternatives
    - Input: a preference profile (a ranking of alternatives by each voter)
    - Output: winning alternative from A
    - Also discussed the idea of deriving a *consensus ranking* over A
  - Different voting rules (Plurality, Borda, approval, STV, etc.) and properties
    - Different rules give different results on same profiles!



Plurality Voting

- *Plurality voting:*
  - **Input:** rankings of each voter
  - **Winner:** alternative ranked 1st by greatest number of voters
    - number of 1st-place rankings is *a*'s *plurality score*
    - *complete* rankings not needed, just votes for most preferred alternatives
    - we'll ignore ties for simplicity
  - This is a most familiar scheme, used widely:
    - locally, provincially, nationally for electing political representatives
  - With only 2 alternatives, often called *majority voting*

- Example preference profile (three alternatives):
  - $A \succ B \succ C$: 5 voters
  - $C \succ B \succ A$: 4 voters
  - $B \succ C \succ A$: 2 voters
- Winner: A wins (plurality scores are A: 5; C: 4; B:2)

The Borda Rule

- *Borda voting rule:*
  - **Input:** rankings of each voter
  - *Borda score* for each alternative *a*: *a* gets m-1 points for every 1st-place rank, *m-2* points for every 2nd-place, etc.
  - **Winner:** alternative with highest Borda score
  - Used in sports (Heismann, MLB awards), variety of other places
  - Proposed by Jean-Charles, chevalier de Borda in 1770 to elect members to the French Academy of Sciences (also Ramon Llull, 13th century)

- Example profile (three alternatives, positional scores of 2, 1, 0):
  - $A \succ B \succ C$: 5 voters
  - $C \succ B \succ A$: 4 voters
  - $B \succ C \succ A$: 2 voters
- Winner: B wins (Borda scores are: B: 13; A: 10; C: 10)
  - Notice: more sensitive to *the entire range of preferences* than plurality is (which ranked *B* last)

Approval Voting

- Approval Voting
  - **Input:** voters specify a *subset* of alternatives they "approve of"
  - Approval score: a point given to a for each approval
    - variant: *k*-approval, voter lists exactly *k* candidates
  - **Winner:** alternative with highest approval score
  - Used in many informal settings (at UN, Doge of Venice, …)
  - Steven Brams a major advocate (see Wikipedia article)

- Example profile (three alternatives, approvals in bold):
  - **A** ≻ B ≻ C: 5 voters (approve of only top alternative)
  - **C** ≻ B ≻ A: 4 voters (approve of only top alternative)
  - **B** ≻ **C** ≻ A: 2 voters (approve of top two alternatives)
- Winner: C wins (approval scores are: C: 6; A: 5; B: 2)
  - Notice: can't predict vote based on ranking alone!

Positional Scoring (Voting) Rules

- Observe that plurality, Borda, *k*-approval, *k*-veto are all each *positional scoring rules*
- Each assigns a *score* $\alpha(j)$ to each rank position j
  - almost always non-increasing in j
- The winner is the candidate *a* with max total score: $\sum_i \alpha(r_i(a))$

In general:	$\alpha(1)$	$\alpha(2)$	$\alpha(3)$	$\alpha(4)$
Plurality:	1	0	0	0
Borda:	3	2	1	0
2-Approval:	1	1	0	0
Veto:	1	1	1	0
and another:	10	2	0	0

Which of these is Better?

- Notice that on the same vote profile, plurality, Borda, and approval gave different winners!

- Which is best?
  - hard to say: depends on social objective one is trying to meet
  - common approach: identify *axioms/desirable properties* and try to show certain voting rules satisfy them
    - we will see it is not possible in general!
- Note: all these voting rules have to have some tie breaking breaking rule. In some cases, that rule is simply a flip of the coin. See the tie vote in a recent election in Mississippi. Even with a large number of voters ties can happen.
- Let's now look at a few more voting rules to get a better sense of things.

There are Hundreds of Voting Rules

- *Single-transferable vote (STV)* or *Hare* system
  - Round *1*: vote for favorite candidate; eliminate candidate with lowest plurality score;
  - Round *t*: if your favorite eliminated at round *t-1*, recast vote for favorite remaining candidate; eliminate candidate with lowest plurality score
  - Round *m-1*: winner is last remaining candidate
    - terminate at any round if plurality score of top candidate is at least *n/2* (i.e., there is a majority winner)
  - Used: Australia, New Zealand, Ireland, recent NDP convention

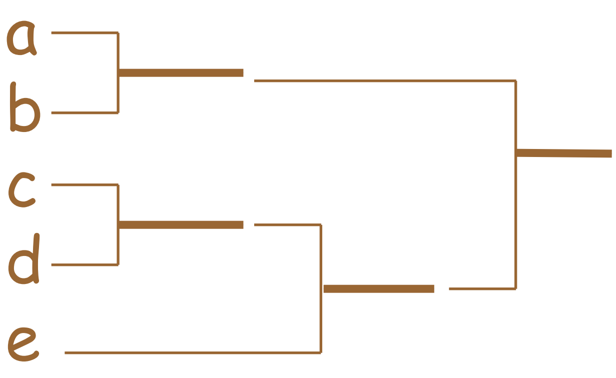
    Needn't be online: voters can submit rankings once
  - When would this be a bad voting rule?
- *Nanson's rule*
  - Just like STV, but use Borda score to eliminate candidates

There are Hundreds of Voting Rules

- *Egalitarian (maxmin fairness)*
  - Winner maximizes minimum voter's rank: $argmax_a \; min_j \; (m\text{-}r_j\,(a))$

- *Copeland*
  - Let $W(a,b,\mathbf{r}) = 1$ if more voters rank $a \succ b$; 0 if more $b \succ a$; ½ if tied
  - Score $s_c(a,\mathbf{r}) = \sum_b W(a,b,r)$; winner is *a* with max score
    - *i.e., winner is candidate that wins most pairwise elections*

- *Tournament/Cup*
  - Arrange a (usually balanced) tournament tree of pairwise contests
  - Winner is last surviving candidate
  - We'll discuss this in more detail later



Condorcet Principle

- How would you determine "societal preference" between a pair of alternatives *a* and *b*?
- A natural approach: run a "pairwise" majority vote: if a *majority* of voters prefer *a* to *b*, then we say *the group prefers a to b*
- *Condorcet winner:* an alternative that beats every other in a pairwise majority vote
  - proposed by Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet in 1785
  - if there is a Condorcet winner, it must be unique
  - a rule is *Condorcet-consistent* if it selects the Condorcet winner (if one exists)

- Condorcet winners need not exist (next slide)
  - Moreover, many natural voting rules are not Condorcet consistent (e.g., plurality, Borda, STV are not), but some are: Nanson, Copeland, Cup, etc.

Condorcet Paradox

- *Condorcet paradox:*
  - suppose we use the pairwise majority criterion to produce a societal preference ranking
  - pairwise majority preferences may induce *cycles* in societal ranking (i.e., the preference relation is not transitive)
- Simple example:
  - $A \succ B \succ C$: *m/3* voters
  - $C \succ A \succ B$: *m/3* voters
  - $B \succ C \succ A$: *m/3* voters
  - Societal ranking has $A \succ B$, $B \succ C$, and $C \succ A$ (!)
  - No clear way to produce a consensus ranking
  - Also evident that this preference profile has no Condorcet winner

Violations of Condorcet Principle

- Plurality violates Condorcet
  - 499 votes: $A \succ B \succ C$
  - 3 votes: $B \succ C \succ A$
  - 498 votes: $C \succ B \succ A$
  - plurality choses A; but B is a CW (B>A 501:499; B>C 502:498)

- Borda violates Condorcet
  - 3 votes: $A \succ B \succ C$
  - 2 votes: $B \succ C \succ A$
  - 1 vote: $B \succ A \succ C$
  - 1 vote: $C \succ A \succ B$
  - Borda choses B (9 pts) ; but A is a CW (A>B 4:3; A>C 4:3)
  - notice *any* scoring rule (not just Borda) will choose B if scores strictly decrease with rank

The Axiomatic Method

- Considerable work studies various *"axioms"* or principles that we might like voting rules to satisfy and asks whether we can devise rules that meet these criteria
- For example, the Condorcet principle is an axiom/property we might consider desirable. We've seen some standard voting rules satisfy it, and others do not.
- Let's consider a few more rather intuitive axioms…

Weak Mononoticity

- *Weak monotonicity:* Let *V* be a set of vote profiles and let *V'* be identical to *V* except that one alternative *a* is ranked higher in some of the votes. Then if *a* is the winner under voting rule r with profile *V*, it should also be the winner with profile *V'*.
  - That is, if *a* is the winner under some voting rule given some voter preferences, then a should remain the winner if a few voters raise their ranking of *a*, but everything else is unchanged.

- STV violates weak monotonicity
  - 22 votes: $A \succ B \succ C$
  - 21 votes: $B \succ C \succ A$
  - 20 votes: $C \succ A \succ B$
  - A wins (C, then B eliminated)…
  - … but if anywhere from 2 to 9 voters in the BCA group "promote" A to top of their rankings, C wins (B, then A eliminated)
- Lot of rules satisfy weak monotonicity (e.g. plurality, Borda, …)

Independence of Irrelevant Alternatives (IIA)

- *Independence of Irrelevant Alternatives (IIA):* Suppose *V'* is a vote profile that is different than *V*, but every vote in *V'* gives the same relative ordering to *a, b*, as it does in *V*. Then if *a* is the winner under a voting rule *r* given profile *V*, the *b* cannot be the winner under profile *V'*.
  - In other words, if the votes are changed, but the *relative* (pairwise) preference for *a* and *b* are identical for every voter, then we can't change the winner from *a* to *b.*

- Borda violates IIA (as do quite a few other voting systems):
  - 3 votes: A ≻ B ≻ C ≻ D ≻ E
  - 1 vote: C ≻ D ≻ E ≻ B ≻ A (switch: C ≻ B ≻ E ≻ D ≻ A)
  - 1 vote: E ≻ C ≻ D ≻ B ≻ A (switch: E ≻ C ≻ B ≻ D ≻ A)
  - C wins under red votes (Borda scores: C:13, A:12, B:11, D:8, E: 6)
  - … but with the blue switches, B wins (scores B:14, C:13, …).
  - Winner from C to B, despite all paired B,C prefs identical in both cases.

Independence of Irrelevant Alternatives (IIA)

- Another view of IIA: suppose *a* wins over *b* in an election. Then we add a new alternative. *Without changing anyone's relative preferences for a and b*, suddenly *b* can win.

- Consider the following preferences:
    - 100 votes: Bush $\succ$ Gore $\succ$ Nader
    - 12 votes: Nader $\succ$ Gore $\succ$ Bush
    - 95 votes: Gore $\succ$ Nader $\succ$ Bush
  - Run a plurality election with only two candidates, Bush and Gore
    - Gore wins over Bush (plurality score of 107 to 100)
  - At the least minute, Nader enters the race:
    - Bush wins the election now (plurality score of 100 to 95 to 12)

Other Principles

- *Unanimity:* if all $v \in V$ rank a first, then a wins
  - relatively uncontroversial
- *Weak Pareto:* if for all $v \in V$ rank $a \succ b$, then b cannot win
  - relatively uncontroversial
- *Non-dictatorial:* there is no voter k s.t. a is the winner whenever k ranks a first (no matter what other voters say)
- *Anonymity:* permuting votes within a profile doesn't change outcome
  - e.g., if all votes are identical, but provided by "different" voters, result does not change (can't depend on voter's identities)
  - implies non-dictatorship
- *Neutrality:* permuting alternatives in a profile doesn't change outcome
  - i.e., result depends on relative position of an alternative in the votes themselves, not on the identity of the alternative
  - implies non-imposition (i.e. every possible ranking is achievable)

Arrow's Theorem

- So can we satisfy all (or even some of these axioms)?
- Arrow's Theorem (1951): Assume at least three alternatives. No voting rule can satisfy IIA, weak Pareto, and non-dictatorship.
  - Most celebrated theorem in social choice
  - Broadly (perhaps too broadly) interpreted as stating there is no good way to aggregate preferences
  - Key point: Arrow's Theorem is phrased in terms of a rule producing a ranking.
- There are a wide variety of alternative proofs
  - Easley and Kleinberg provide one proof (see Ch. 23.11)
  - An especially simple proof is given the next two slides for anyone interested.