CSC200: Lecture 30

Announcements

- Assignment 3 is due this Wednesday, start of class.
- Term test 2 will take place Friday, February 12 in the lecture room.
- Scope will include some or all of the following topics: matching markets, stable matchings, hubs and authorities/page rank algorithms for ranking pages, VCG and GSP applied to adwords auctions.
- I will not include cascades (chapter 16) and direct benefit effects (chapter 17) in this term test but they will surely be on the next (and final assignement) as well as probably on the next quiz and the final exam.

Todays agenda

- Review and finish discussion Chapter 17 (direct benefit effects)

Note

The goal is to qualitatively understand the dynamics of market demand in a stylized (but still interesting) setting.

  - Some mathematics is needed to get a clearer "picture" but the **"big picture"** is what is important here.
  - What kind of **conclusions** can be inferred from such an understanding?

- For whatever time is left, remaining part of today will be a tutorial session as the TAs and I have been receiving questions relating to the assignment that is due Wednesday.

Lets review the big "picture" (but first in words) for market demand dynamics

We are considering the demand for a good, participation in a club, subscriptions to a newspaper, etc. where we assume an unlimited supply of the good being produced, offered, etc. at some fixed price p.

- Individual x in $[0, 1]$
- Each x has an intrinsic value $r(x)$ (in text called reserve price) for participating and we assume the function $r(x)$ is continuous and strictly decreasing.
- Interesting prices satisfy $r(0) > p > r(1)$ and that will be the assumption.
- The (overall) value that x has for participating also depends on the current demand (level of participation) z. This is defined (for the purposes of this chapter) as the product $r(x)f(z)$ where f is increasing.

Summarizing the model

- Given a fixed price p^*, and a current market demand (participation level) being a fraction z of current users, an individual will want to participate if and only if their overall value $r(x)f(z)$ is at least p^*.
- We first assume that $f(0) = 0$ so that we assume there is no one willing to participate if there is no current participation.
- Since function r has maximum value $r(0)$, for any z, there will not be any participation if $r(0)f(z) < p^*$.
- So starting at some demand level $z = z_0$ (with $r(0)f(z)$ at least p^*) our big picture goal will be to understand how the market demand will evolve over time.

What if everyone makes a perfect (shared) prediction?

Assumption

If everyone makes the same prediction about the fraction z buying the good, and then every consumer x decides to buy based on whether or not $r(x)f(z)$ is at least p^*, then (eventually) the fraction of adopters will actually be this z.

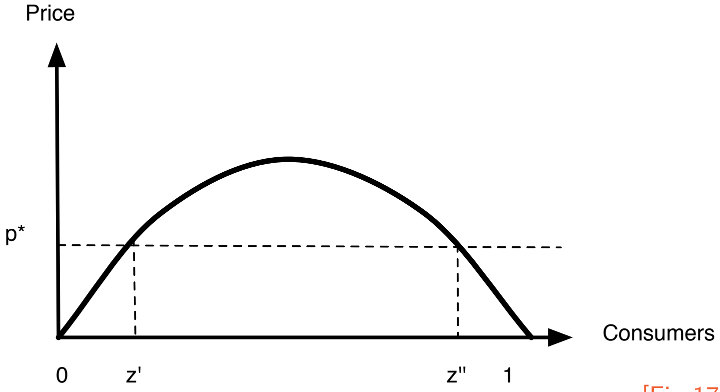
- This is called a self-fulfilling expectations equilibrium for the quantity z (at price $p^* > 0$).
- For a fixed z, as x increases, $r(x)f(z)$ decreases, so we have:

Fact

If $p^* > 0$ and z in $(0, 1)$ is a self fulfilling expectations equilibrium at p^*, then $p^* = r(z)f(z)$. By the assumption that $f(0) = 0$, $z = 0$ is also a self-fulfilling expectations equilibrium.

Recalling the case of $r(x) = 1 - x$ and $f(z) = z$

- As an example of the model, the text considers the decreasing reserve price (intrinsic value) function $r(x) = 1 - x$ and the increasing direct benefit function $f(z) = z$.
- Then in addition to $z = 0$, a self-fulfilling expectations equilibrium $z > 0$ must satisfy $p^* = (1 - z)z$ for a given price p^*.



[Fig 17.3, E&K]

What if beliefs about market demands are not correct?

- Changing the story (but not the model), assume that we are now tracking participation in a given activity, say a large online social network or involvement in a political movement.
  - We view such participation as being more fluid than buying an object unless the cost is minor.
  - In the case of participation there are also maximum costs p^* (monetary or effort or reputation etc which can all be seen to be ultimately "costs") that a person will pay.
  - Note that some online dating services do charge a fixed (say monthly) fee to belong. And it could be that services like Linkedin of other social media sites will also start charging.

- We maintain the same model that x will participate if and only if $r(x)f(z)$ is at least p^*.

Discrete step dynamics

- We will assume that at some initial time $t = 0$, the observable demand level is z_0. This will cause the demand to change to some z_1 at time $t = 1$ and similarly the demand then changes to z_2 at time $t = 2$, etc.
- That is, if everyone observes demand z at some point of time, then the set of people participating at the next time step will be all those people x in $(0, \hat{z}]$ where $\hat{z}$ satisfies $r(\hat{z})f(z) = p^*$.
- That is, the next demand level will be

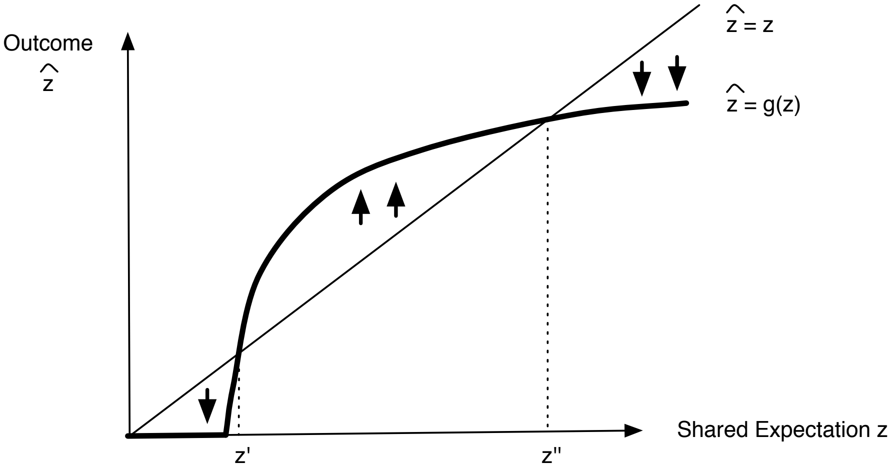
$$\hat{z} = g(z) = r^{-1}\left(\frac{p^*}{f(z)}\right)$$

Since $r()$ is continuous and decreasing, such a solution will exist as long as $p^*/f(z)$ is at most $r(0)$. Otherwise $g(z) = 0$.

The same specific case of $r(x) = 1 - x, f(z) = z$

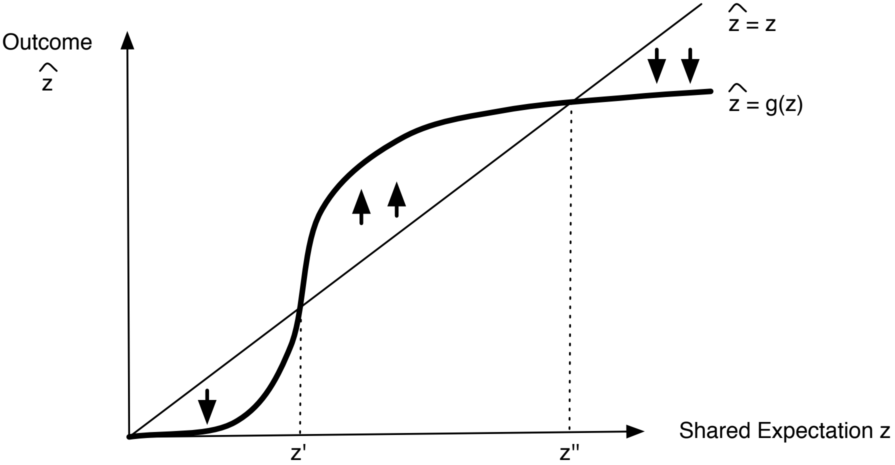
- For definiteness we will again take the specific case of $r(x) = 1 - x$ and $f(z) = z$. Hence $r(0) = 1$ and $p^*/f(z) = p^*/z$. So we want the shared demand observation z to satisfy $p^*/z = p^*/f(z) \leq r(0) = 1$ or equivalently that z is at least p^*.
- It is easy to verify that $r^{-1}(y) = 1 - y$
- Hence in this case, $g(z) = r^{-1}(\frac{p^*}{z}) = 1 - \frac{p^*}{z}$ when $z \geq p^*$, and $g(z) = 0$ otherwise.
- So here again is the picture for this specific case.

Change in demand for $r(x) = 1 - x$, $f(z) = z$



[Fig 17.5, E&K]

We would expect to see a smoother convergence below the tipping point



[Fig 17.6, E&K]

Discrete step dynamic behaviour

Starting at some initial observable demand z_0, we generate future demands according to $z_{t+1} = g(z_t)$ for each time step $t = 0, 1, 2, \ldots$

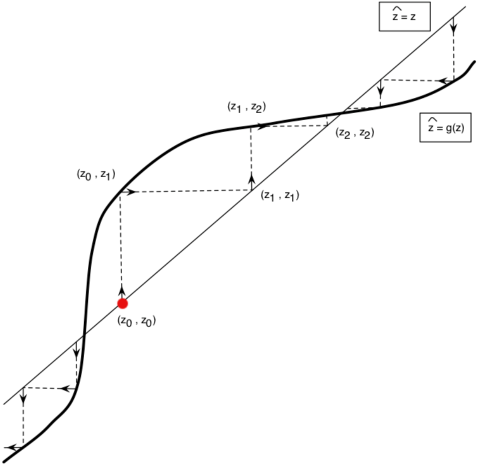


Figure : The $g(z)$ curve. [Fig 17.9, E&K]

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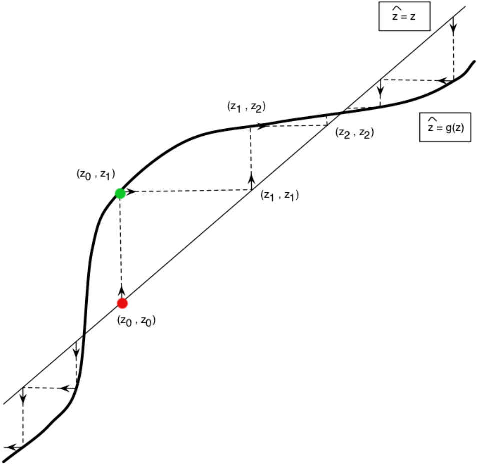

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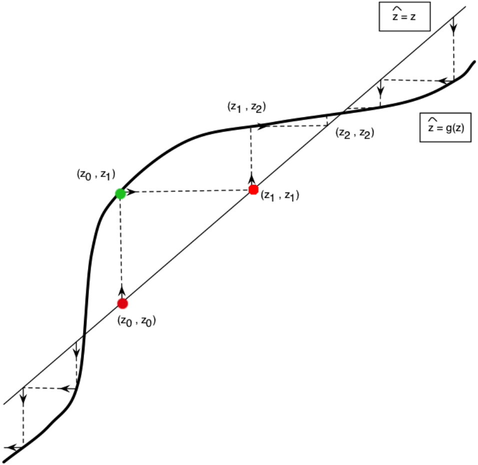

Figure : The $g(z)$ curve. [Fig 17.9, E&K]

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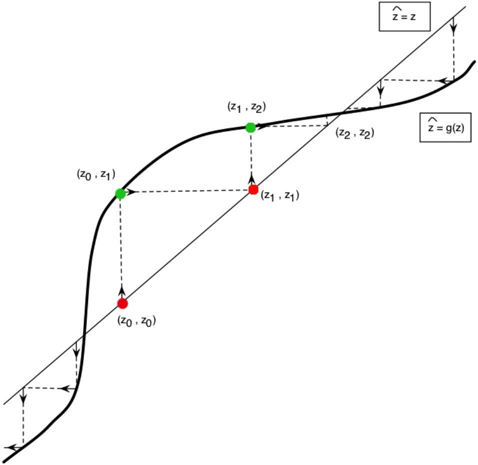

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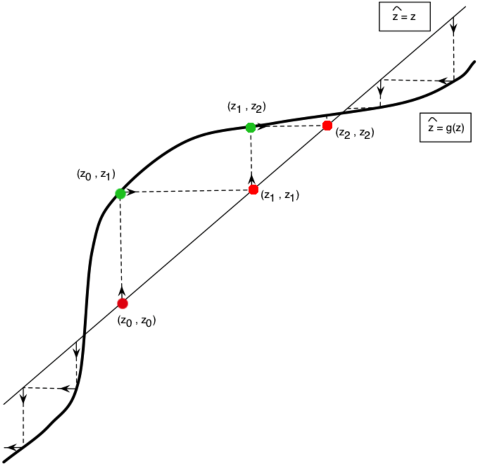


Figure : The $g(z)$ curve. [Fig 17.9, E&K]

Equilibrium Points

- The previous picture show a particular sequence of demand levels converging to a point z . Such a point is an equilibrium point (what the text calls a self-fulfilling expectations equilibrium).

- Recall how such an equilibrium is defined?

A demand level z is an equilibrium if when everyone makes the same prediction about fraction z participating, and every consumer x decides based on whether or not $r(x)f(z)$ is at least p^*, then the demand fraction will continue to be z.

- Once we establish what are the equilibrium points, the dynamic discrete step picture allows us to visualize

if and how we would converge to any such equilibrium for the process determined by functions r and f given how the g in $z_{t+1} = g(z_t)$ is determined by functions r and f

Recalling the definition of g and dynamic step behaviour

- We have
  - At some initial time $t = 0$, the observable demand level is z_0.
  - For any t, $z_{t+1} = g(z_t)$
- That is, if everyone observes demand z at some point of time, then the set of people participating at the next time step will be all those people x in $(0, \hat{z}\,]$ where $\hat{z}$ satisfies $r(\hat{z})f(z) = p^*$.
- That is, the next demand level will be

$$\hat{z} = g(z) = r^{-1}\left(\frac{p^*}{f(z)}\right)$$

- Such a solution will exist as long as $p^*/f(z)$ is at most $r(0)$. Otherwise $g(z) = 0$.

Recalling g for the specific $r(x) = 1 - x, f(x) = x$?

- For the specific case of $r(x) = 1 - x$ and $f(z) = z$ we have
  1. $r(0) = 1$
  2. $r^{-1}(y) = 1 - y$ and
  3. $p^*/f(z) = p^*/z$.

- So in this case

$$g(z) = r^{-1}\left(\frac{p^*}{z}\right) = 1 - \frac{p^*}{z}$$

- For such a solution $g(z)$, we need $p^*/f(z) = p^*/z$ is at most $r(0) = 1$ so that z has be at least p^*; else $g(z) = 0$. (This is a cold start problem of how to achieve a sufficient initial demand and we will return to this issue.)

Some more qualitative comments

- Dynamic demand change is the result of a myopic best response by all individuals at to whether or not to continue to participate.
  - With direct benefit effects, equilibria are typically not socially optimal. Why? And what is social welfare in this setting?

- The marketing of any product or membership will not succeed if we do not get past the tipping point. Here we are assuming $f(0) = 0$ and we will see the situation changes when $f(0) > 0$.

- All things being equal, being the product first to tipping point is very important. But all things are not equal and markets change. Producers can sometimes use established reputation (branding) along with special offers and new features to overtake early market control.

- For the specific example we have been considering there is only one stable equilibrium and when we can get past the unstable z' the demand will move to the stable equilibrium z''.
  - It makes sense to initially have low (maybe unprofitable) prices and even give away things initially.

The tragedy of the "anti-commons"

- There is a well studied phenomena (see section 24.2 of the text) called "The tragedy of the commons" so named in a 1968 article by Hardin based on 1833 essay by LLoyd.
- Wikipedia defines the phenomena as follows: The Tragedy of the commons is an economic theory which states that individuals acting independently and rationally according to each's self-interest, behave contrary to the best interests of the whole group, by depleting some common resource.
- The fact that individuals act myopically (in assuming that todays demand for a resource will be tomorrows demands) leads to a similar but different phenomena that we can call "The tragedy of the *anti-commons*".
- That is, the collective low-usage of a resource (i.e., the opposite of depletion) can also lead to an overall decrease in social welfare.
- Here , social welfare is defined as the sum of all individual reservation prices (for those who purchase or subscribe) minus the total price of producing the good.

How do things change when $f(0) > 0$?

- We are still assuming $f(z)$ is increasing but now don't have to be as concerned with the "cold start" problem; that is, individuals may have some minimum value independent of other users.
- The text now considers the same $r(x) = 1 - x$ and a different $f(z) = 1 + az^2$ so that now we always have $f(z) \geq 1$ and assuming $0 < p^* < 1$, we have

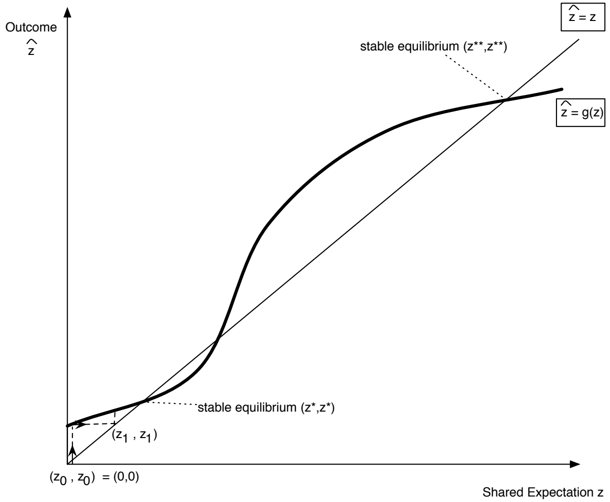
  $$1 = r(0) > p > r(1)$$

  Therefore, we will always have a solution since $p^*/f(z) < 1$.
- Hence as before we have

  $$g(z) = r^{-1}\left(\frac{p^*}{f(z)}\right) = 1 - \frac{p^*}{1 + az^2}$$

- A market of zero users is no longer an equilibirum! Even if everyone believes there might not be any other users, some perople will still want to purchase because of their own intrinsic value for the item.

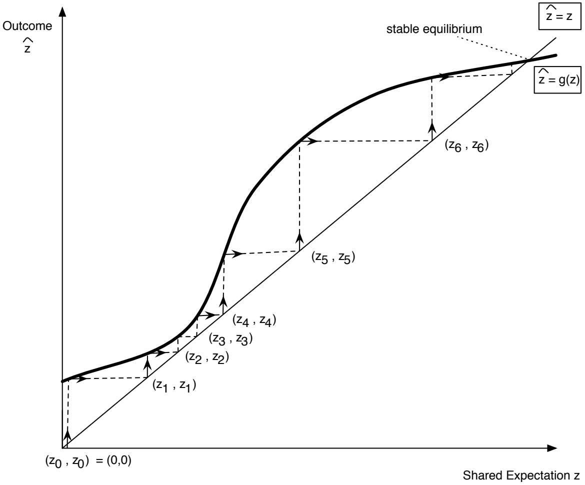
New dynamics: From zero demand to a small equilibirum demand



[Fig 17.11, E&K]

Lowering the price to some q^*

- Bottleneck becomes small passageway
- Now we have $g(z) = r^{-1}\left(\frac{q^*}{f(z)}\right) = 1 - \frac{q^*}{1+az^2}$



[Fig 17.13, E&K]