CSC200: Lecture 29

Announcements

- Quiz this Friday, February 5
- Quiz is on VCG/GSP for search engine auctions
- Term test Friday, February 12; scope includes all topics following term test 1 up to and including chapter 16 (but not chapter 17).
- **BAD TYPO IN SLIDES FOR L18 and L19**
  In slide 24 of Lecture 18 (and the same slide appearing as slide 5 in Lecture 19) there was a bad typo. Namely, in the bottom left figure, the caption said "Increase price of C1,C2 (or C2)" implying that C2 was a constricted set which it is not. The slides have been corrected. I apologize for that error and thank the student(s) who brought it to our attention on the discussion board.
- Today's agenda:
  1. Continue Chapter 17 (direct benefit effects)

Markets with a huge number of consumers

The assumption throughout Chapter 17

Any single individual won't affect the aggregate behaviour of the market.

- That is, whether or not I buy a few shares of a particular stock will not impact prices or overall demand.
- But if many people want to buy or sell a given stock then prices will be impacted which in turn will impact further demand.

- A common way to deal with a huge but finite system of individuals is to abstract the system as if individuals are just points on say the real line segment $[0, 1]$.

- Then each individual has no "mass" but subintervals do have proportional mass.

Consumers as points on the line $[0, 1]$

- We now assume

each consumer is a point on the line segment $[0, 1]$
wanting to buy one unit of a good.

- We also assume the consumer's willingness to buy the item depends both on
  1. their intrinsic interest in (i.e. value for) the item and
  2. the number of other people using the good (i.e. the direct benefit effect); the more users the more the item is worth.

- To start, the text first assumes no direct benefit effect and then studies how direct benefits change things.

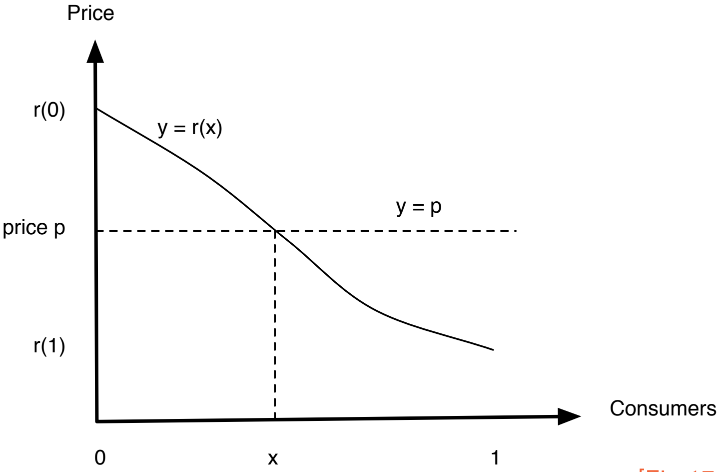
Intrinsic interest, the reservation price

- We let the intrinsic interest be specified by a single reservation price $r(x)$ for individual x in $[0,1]$. An individual x will buy the item at a price p if and only if $r(x)$ is at least p.
- Without loss of generality, we arrange the individuals so that
$$x < y \text{ implies } r(x) \geq r(y)$$
- For analysis we further assume $x < y$ implies $r(x) > r(y)$ and also that r is a continuous function on $[0,1]$. (Once we have made the abstraction to the real line these are not critical assumptions.)
- It follows that (except for the single point $x = 0$)

1. no one will buy the good at price $r(0)$ or more
2. at a price $r(1)$ or less everyone will buy the good.

Market demand for the good at a given price

- By continuity, for every price p with $r(0) > p > r(1)$, there is a unique x (called the market demand at p) such that $r(x) = p$.
- That is, an x fraction will want to buy a unit of the good at price p.



[Fig 17.1, E&K]

Market with large number of producers

- Now the discussion proceeds to assume that there is some (say industry wide) cost p^* at which a unit of the good can be produced.
  - Perhaps to make this more realistic, assume this cost includes an industry wide small profit/unit
  - In any case we are assuming that no producer is willing to supply the good at price below p^* per unit of good.
- Another (more substantial) assumption:

There are enough producers capable of producing an unlimited supply of the good and no single producer can change the market. Implicitly the goods are identical, independent of the producer.

- Thus in aggregate these producers can supply as much of the good as desired at price p^* per unit but will not produce any of the good at price below p^* per unit.
- This also fixes the price at p^* since by assumption competition will not allow any producer to ask for more than p^* per unit.

Equilibrium quantity of good (at p^*)

- This fixed (non negotiable) cost of p^* per unit, which we can assume to be between $r(0)$ and $r(1)$, determines a unique x^* such that $r(x^*) = p^*$.

- This x^* is an equilibrium in the overall consumption of the good in the following sense.
    - If less than a fraction x^* purchased the good, there would be excess consumers with reservation prices above p^* and hence they would want to buy the good and thereby drive up consumption.
    - By assumption, consumers will not pay more than their reservation price meaning that it is not sustainable to have more than a fraction x^* of purchasers.

And now what happens with the addition of direct benefit effects?

- We are assuming that having a large fraction of existing users of the good makes the good that much more desirable.

- This is modeled by now saying that the reservation price for consumer x is $r(x)f(z)$ when there is a fraction z of current users for some function $f(z)$ that is increasing in z.

- A few more assumptions, mainly to simplify the discussion; namely, assume f is continuous and $r(1) = 0$. And for now also assume $f(0) = 0$.

- So now a consumer x is willing to buy a unit of the good at price p^* if x believes a fraction z of users will also be using the good and $r(x)f(z)$ is at least p^*.

What if everyone makes a perfect (shared) prediction?

Self-fulfilling expectations equilibrium

If everyone makes the same prediction about the fraction z buying the good, and then every consumer x acts on this assumption and decides to buy based on whether or not $r(x)f(z)$ is at least p^*, then (eventually) the fraction of adopters will actually be this z.

- This z is called a self-fulfilling expectations equilibrium for the quantity z (at price $p^* > 0$).
- For a fixed z, as x increases, $r(x)f(z)$ decreases, so we have:

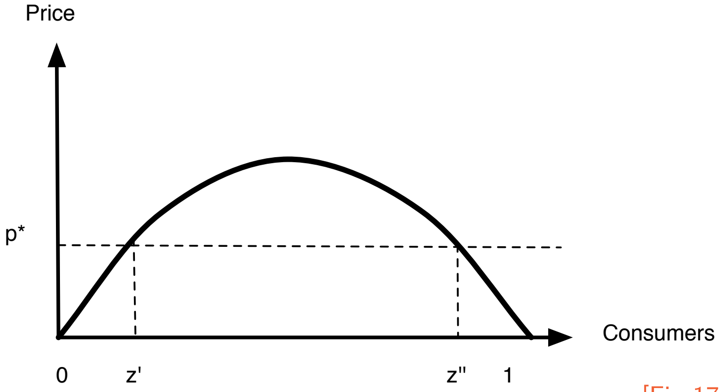
Fact

If $p^* > 0$ and z in $(0, 1)$ is a self fulfilling expectations equilibrium at p^*, then $p^* = r(z)f(z)$. Why? By the assumption that $f(0) = 0$, $z = 0$ is also a self-fulfilling expectations equilibrium.

- This is a more complex (and more interesting) situation than without direct benefits in which case high prices simply imply low demand.

The concrete case of $r(x) = 1 - x$ and $f(z) = z$

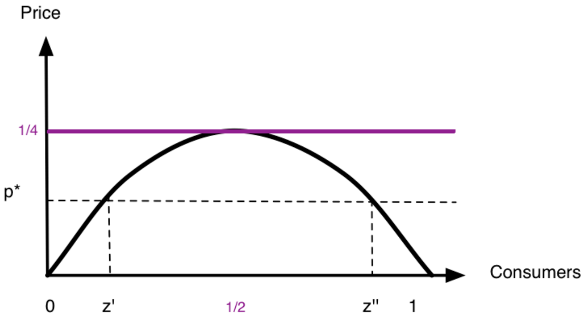
- As an example of the model, the text considers the decreasing reserve price (intrinsic value) function $r(x) = 1 - x$ and the increasing direct benefit function $f(z) = z$.
- Then in addition to $z = 0$, a self-fulfilling expectations equilibrium $z > 0$ must satisfy $p^* = (1 - z)z$.



[Fig 17.3, E&K]

What are the equilibria for this example?

- By taking the derivative of $h(z) = r(z)f(z)$, we see that $h(z)$ has maximum value at $z = \frac{1}{2}$ (and hence $h(z) = \frac{1}{4}$) so that for $p^* > \frac{1}{4}$ there is no (real valued) solution to $p^* = r(z)f(z)$
- The case $p^* = 0$ is not interesting; we will soon consider the special case $p^* = \frac{1}{4}$.
- For any p^* in $(0, \frac{1}{4})$, there are exactly two distinct zeros z', z'' and at the points $z = 0$, z', z'', if everyone believes exactly a z fraction will be buying according to the reservation price, then precisely this fraction will do so.



Why can't there be other equilibria?

- What happens when the demand z is not one of these equilibria points z', z'' (for a price $p^* < \frac{1}{4}$)?

- Three cases:
  1. If $0 < z < z'$, then $r(z)f(z) < p^*$ and there is downward pressure on the demand since the reservation price is less than p^*.
  2. If $z' < z < z''$, then there is upward pressure on demand since $r(z)f(z) > p^*$ and more purchasers are willing to buy.
  3. If $z'' < z$ then we again have $r(z)f(z) < p^*$ causing downward pressure on the demand.

- Note the qualitative difference between z' and z''.
  - Values of z near z'' will push the demand toward z''. That is, z'' is a very stable equilibrium.
  - In contrast, demand predictions around z' are very unstable in that the demand pressure can go either way.

More qualitative comments re equilibria

- The unstable equilibrium point z' is called a critical or tipping point. It is indeed critical for the producers to get past this tipping point in the demand.

- As the price p^* is lowered, the critical point z' (in this reasonably illustrative example) gets lowered and the eventual demand gets larger moving toward demand z''. This is why it is often in the interest of a company to lower initial prices to get past the tipping point.

- We now return to the special case of $p^* = \frac{1}{4}$. Now there is just one non zero equilibrium at $z = \frac{1}{2}$. Following the reasoning for the case of $0 < z < z'$, any deviation from $z = \frac{1}{2}$ will result in downward pressure so that this equilibrium is highly unstable.

What if everyone does not make a perfect (shared) prediction?

- Changing the story as to perfect shared predictions (but not the model), assume that we are now tracking participation in a given activity, say a large online social network, or television series, or involvement in a political movement.
  - We view such participation as being more fluid than buying an object unless the cost for the object is minor.
  - In the case of participation there are maximum costs p^* (monetary or effort or reputation etc which can all be seen to be ultimately "costs") that a person will pay.

- We maintain the same model that x will participate if and only if $r(x)f(z)$ is at least p^*.

Discrete step dynamics

- We will assume that at some initial time $t = 0$, the observable demand level is z_0. This will cause the demand to change to some z_1 at time $t = 1$ and similarly the demand then changes to z_2 at time $t = 2$, etc.
- That is, if everyone observes demand z at some point of time, then the set of people participating at the next time step will be all those people x in $(0, \hat{z}\,]$ where $\hat{z}$ satisfies $r(\hat{z})f(z) = p^*$.
- That is, the next demand level will be the $\hat{z}$ satisfying:

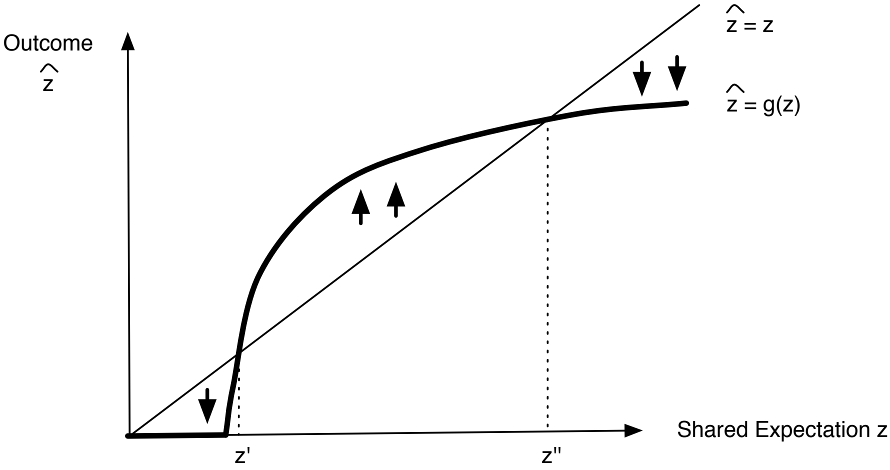
$$\hat{z} = g(z) = r^{-1}\left(\frac{p^*}{f(z)}\right)$$

Since $r()$ is continuous and decreasing, such a solution will exist as long as $p^*/f(z)$ is at most $r(0)$. Otherwise $g(z) = 0$.

The same specific case of $r(x) = 1 - x, f(z) = z$

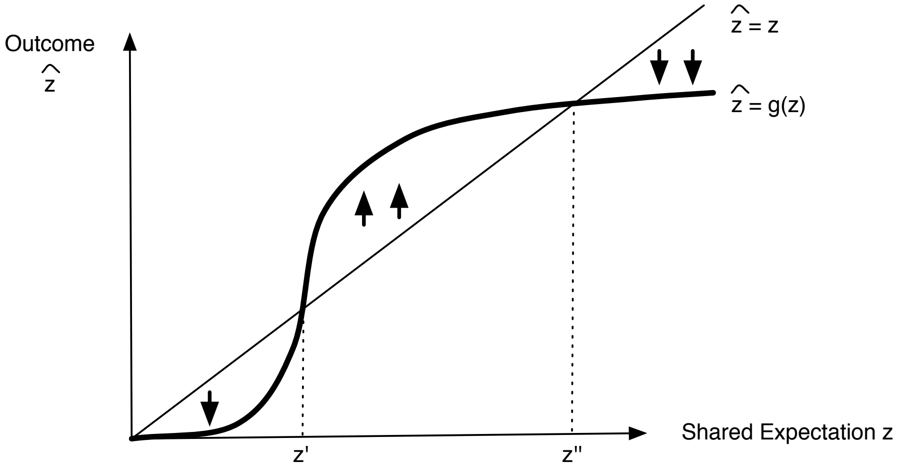
- For definiteness we will again take the specific case of $r(x) = 1 - x$ and $f(z) = z$. Hence $r(0) = 1$ and $p^*/f(z) = p^*/z$. So we want the shared demand observation z to satisfy $p^*/z = p^*/f(z) \leq r(0) = 1$ or equivalently that z is at least p^*.
- It is easy to verify that $r^{-1}(y) = 1 - y$
- Hence in this case, $g(z) = r^{-1}(\frac{p^*}{z}) = 1 - \frac{p^*}{z}$ when $z \geq p^*$, and $g(z) = 0$ otherwise.

Change in demand for $r(x) = 1 - x$, $f(z) = z$



[Fig 17.5, E&K]

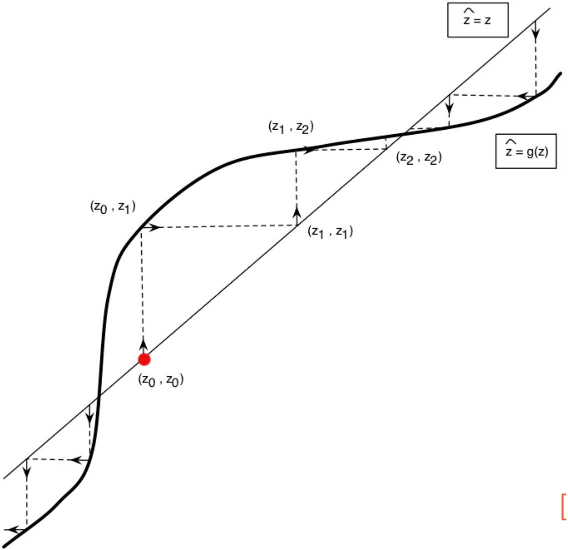
What we expect to see more generally



[Fig 17.6, E&K]

Discrete step dynamic behaviour

Starting at some initial observable demand z_0, we generate future demands according to $z_{t+1} = g(z_t)$ for each time step $t = 0, 1, 2, \ldots$



[Fig 17.9, E&K]

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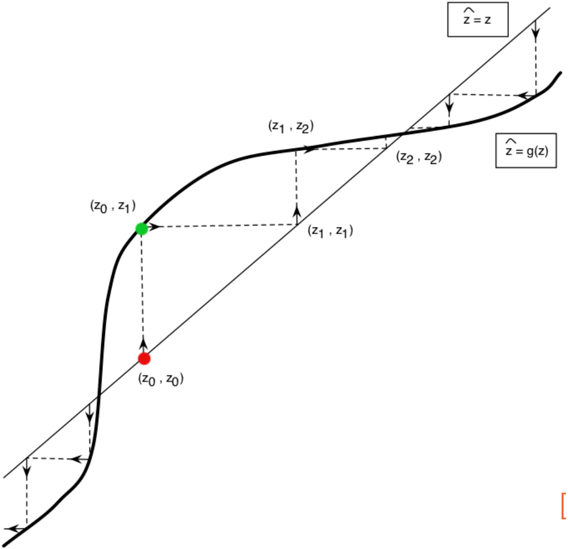

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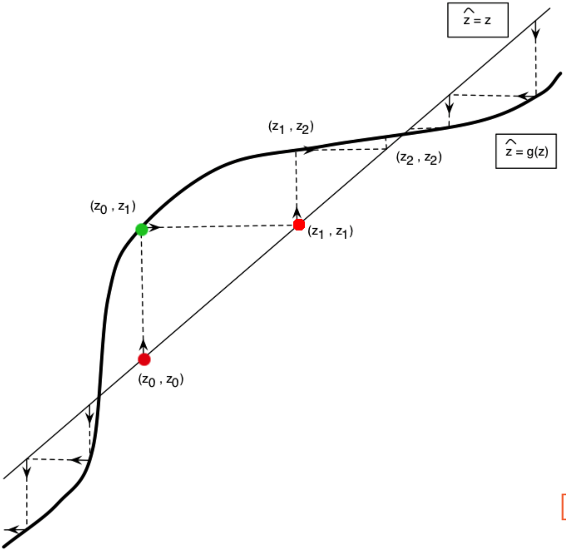

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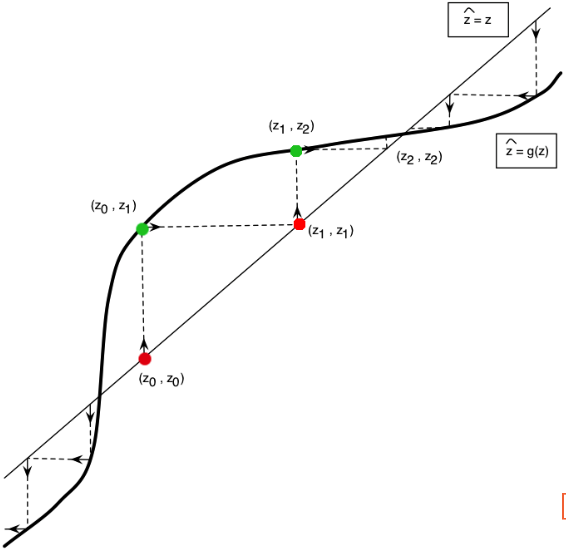

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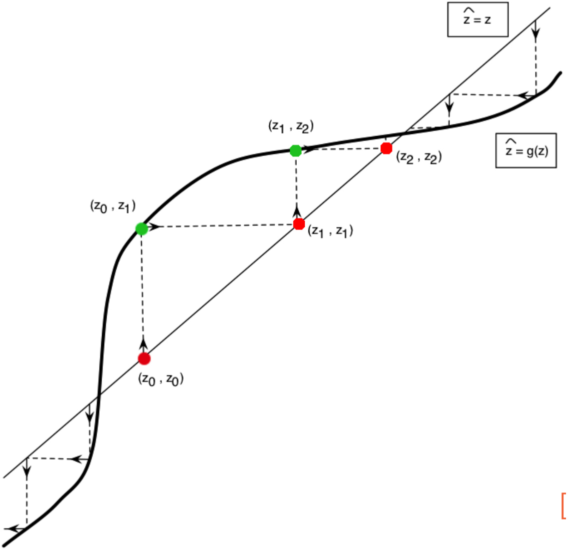

[Fig 17.9, E&K]