CSC200: Lecture 21

- Today
  - kidney exchange; matching in a graph
  - We can view stable matching and kidney exchange problems as mechanism design without money
  - Note: Stable matching and kidney exchange discussion not in text
  - Quick review of fall term

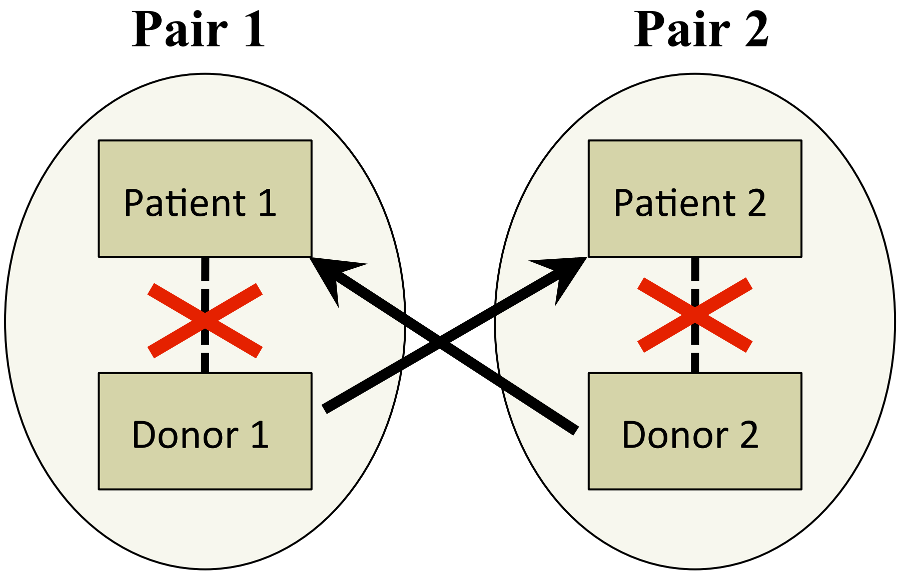
- After the break:
  - An additional lecture by Joanna Drummond on stable matching
  - The internet and search engines: Chapters 13,14
  - internet advertising; return to a matching market application: Ch.15
  - various network phenomena at a more aggregate (population) level (Ch. 16-18), e.g., information cascades, power laws, tipping points, etc.
- Announcements
  - No usual office hours tomorrow; I should be in my office 10AM-1:45PM
  - I may post one or two questions for Assignment 3 dealing with search engines but assignment not due until February 10.

Why Patient Matches are Complicated

- From Sönmez, Ünver. Matching, Allocation, and Exchange of Discrete Resources, *Handbook of Social Economics,* 2009
  - *Blood compatibility test:* There are 4 blood types, O, A, B, and AB. O type kidneys are blood-type compatible with all patients; A type kidneys are blood-type compatible with A and AB type patients; B type kidneys are blood-type compatible with B and AB type patients; and AB type kidneys are only blood-type compatible with AB type patients
  - *Tissue compatibility test (or crossmatch test):* 6 HLA (short for human leukocyte antigen) proteins (3 inherited from the mother and 3 inherited from the father) located on patient and donor DNA helices respectively play two roles in determining tissue compatibility. If antibodies exist in the patient blood against the donor HLA, then the donor kidney cannot be transplanted to the patient and it is deemed tissue-type incompatible. It is reported that, on average, there is only 11% chance of tissue-type incompatibility for a random donor and patient (Zenios, Woodle, and Ross, 2001). Exact HLA match is not required for tissue compatibility; however, there is a debate in the medical literature about how important the closeness of HLA proteins of the patient and donor are for the long-run survival rate of a transplanted kidney.

Live Paired Donation

- What if patient and willing partnered donor are incompatible?
    - Find another pair and swap!
    - Proposed in 1986, realized around 2003



Live Paired Donation

- What if patient and willing partnered donor are incompatible?
  - Find another pair and swap!
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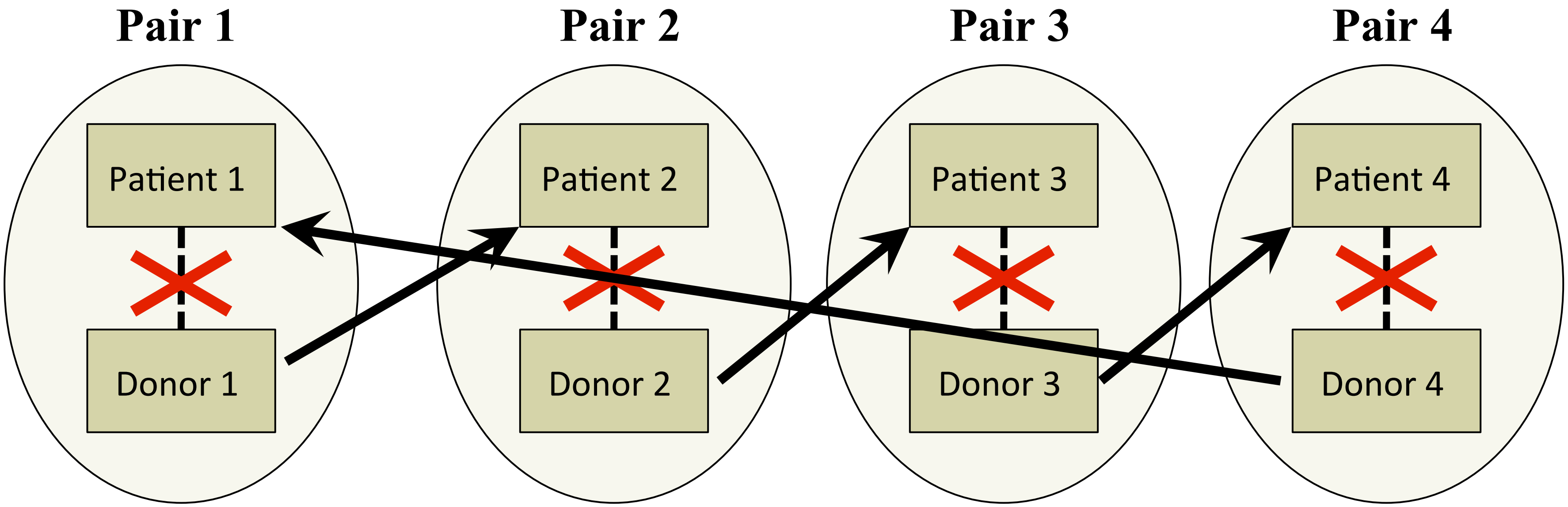
- Why not extend to larger cycles?



The Practicalities

Implementing paired donations a logistical challenge

Incentive issues:

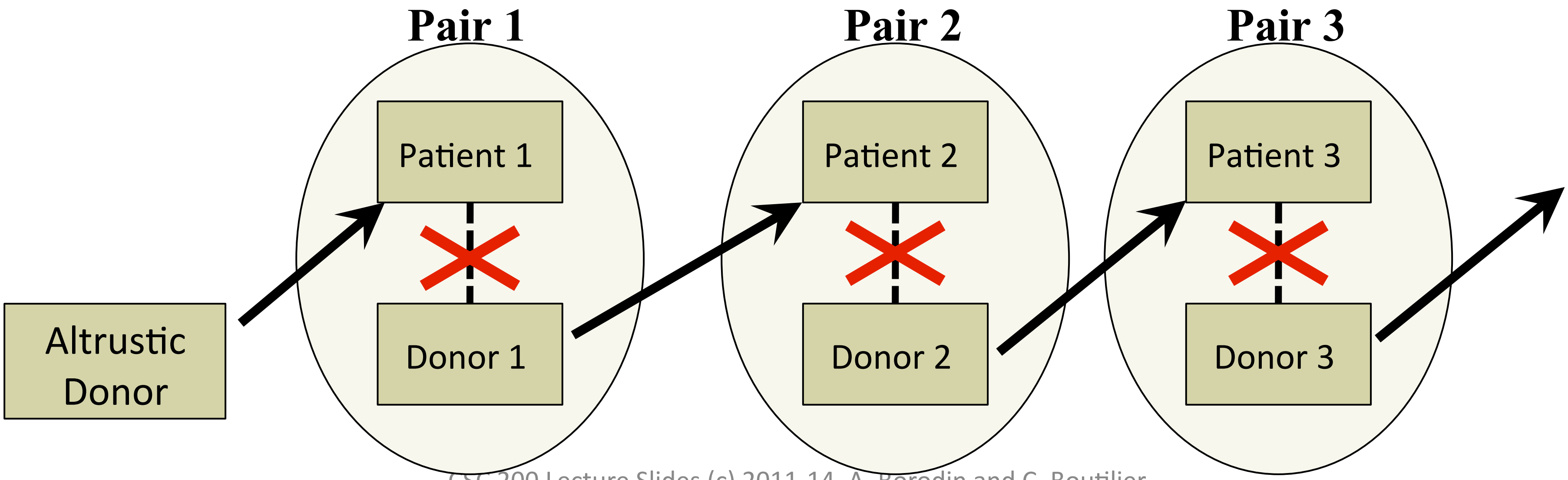
- what if a potential donor reneges (or dies, or gets ill) once his paired patient receives kidney?
- patient waiting for that kidney lost a valuable resource: her paired donor's kidney!
- require that donations and transplants be done simultaneously

So we need 2 (or 3, or *K* for cycle of size *K*) *pairs* of operations going on simultaneously (each transplant requires ***two*** operations: donor+recipient)

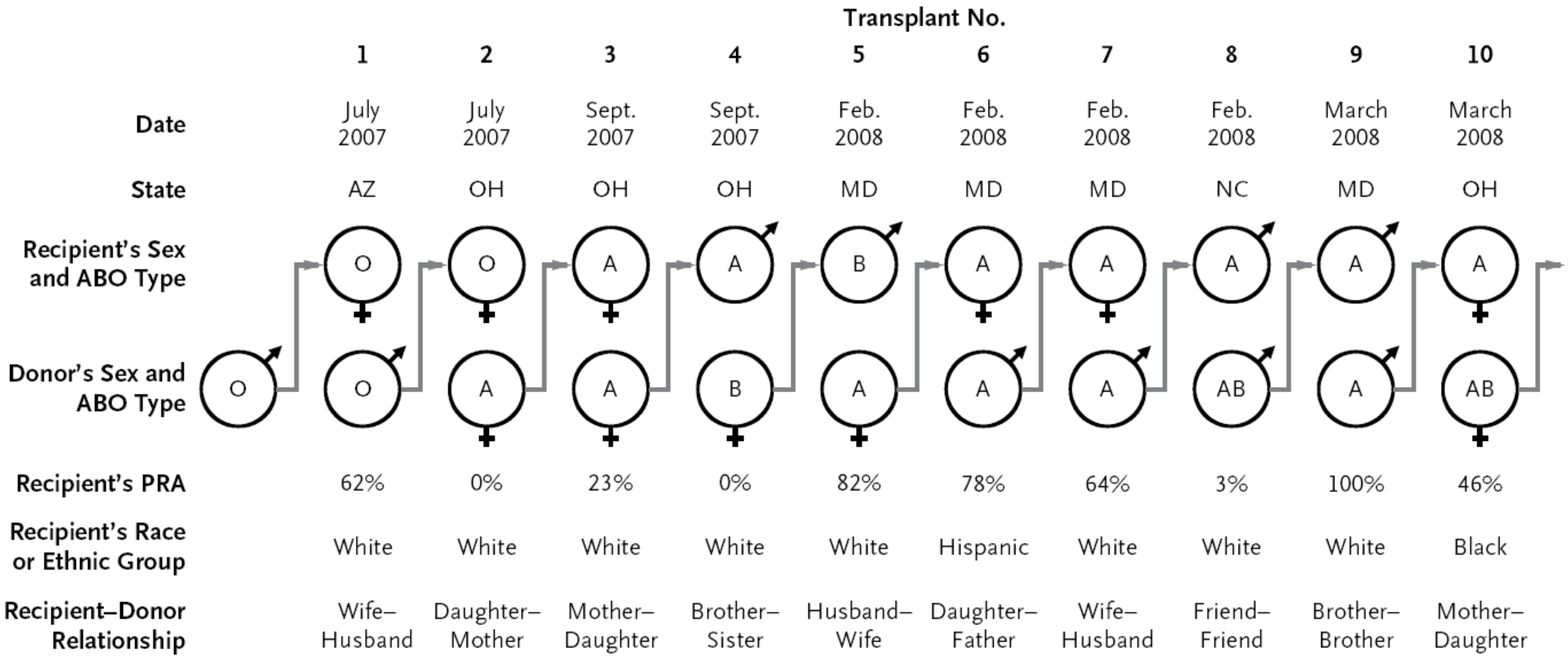
- and live kidneys travel best inside their donors, so must be physically proximal (e.g., same hospital, same city)
  - *e.g., some hospitals will not accept organs transported by air*
- severely limits size of cycles that can be realized in practice

Altruistic Donor Chains

- An incredible innovation
  - a single altruistic donor can mitigate much of the risk of reneging on promises, allowing long chains (*with no need for simultaneity*)
  - each recipient in chain must bring a willing donor to the exchange
  - no donor-patient pair gives up donor kidney before receiving one
  - if chain breaks, no pair has lost their most valuable resource
- Longest chain to date:
  - Chain 124 in US: 30 transplants (60 people): Nov.2011-Feb.2012
  - an 28-pair chain in 2014 (may still be ongoing?)



Never-Ending Altruistic Donor Chains



Transplant No.		1	2	3	4	5	6	7	8	9	10
Date		July 2007	July 2007	Sept. 2007	Sept. 2007	Feb. 2008	Feb. 2008	Feb. 2008	Feb. 2008	March 2008	March 2008
State		AZ	OH	OH	OH	MD	MD	MD	NC	MD	OH
Recipient's Sex and ABO Type		♀ O	♀ O	♀ A	♂ A	♂ B	♀ A	♀ A	♂ A	♂ A	♀ A
Donor's Sex and ABO Type	♂ O	♂ O	♀ A	♀ A	♀ B	♀ A	♂ A	♂ A	♂ AB	♂ A	♀ AB
Recipient's PRA		62%	0%	23%	0%	82%	78%	64%	3%	100%	46%
Recipient's Race or Ethnic Group		White	White	White	White	White	Hispanic	White	White	White	Black
Recipient–Donor Relationship		Wife–Husband	Daughter–Mother	Mother–Daughter	Brother–Sister	Husband–Wife	Daughter–Father	Wife–Husband	Friend–Friend	Brother–Brother	Mother–Daughter

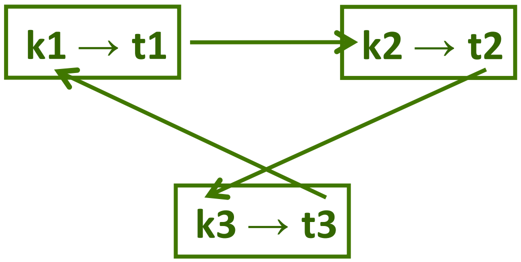
A Nonsimultaneous, Extended, Altruistic-Donor Chain, Rees et al, New England Journal of Medicine, 2009

Modeling Issues

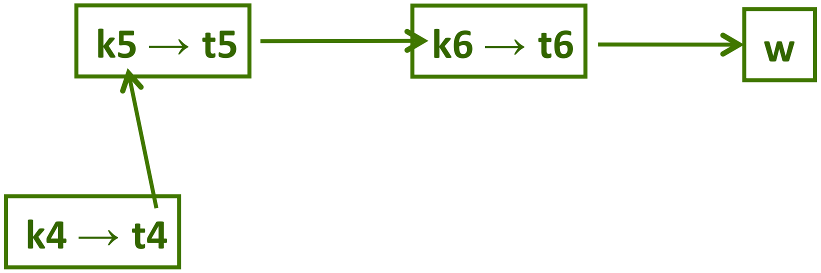
- Model is a "one-sided" matching model:
  - one set of agents: a patient-donor pairs is a single entity
  - each pair has a *preference ordering* over the set of other pairs
    - a pair *p* can get a kidney from any compatible pair *q*
    - *but some kidneys are "more compatible" than others*
  - a pair *p* can never be forced to give up its kidney
    - so it's not a bipartite graph model (in econ, called a "housing market" because everybody comes in owning a kidney, and we're swapping them)
- A *matching* (may not be perfect) finds *"trading cycles"*
  - swapping of kidneys like before, but should be stable
  - other objectives possible (one can use utility measures)
  - can also allow *"chains":* a pair may give up a kidney to get increased priority on the waiting list for cadaveric kidneys

Trading Cycles and Chains

- In the cycle:
  - pair 1: transplant patient *t1*, gets kidney *k2* from donor in pair 2
  - pair 2: patient *t2* gets *k3* from pair 3
  - pair 3: patient *t3* gets *k1* from pair 1
- In the chain:
  - *t4* gets *k5*, *t5* gets *k6*, *t6* gets higher priority on waiting list
    - might convince *k4* for become an altruistic donor



Cycle of Size 3

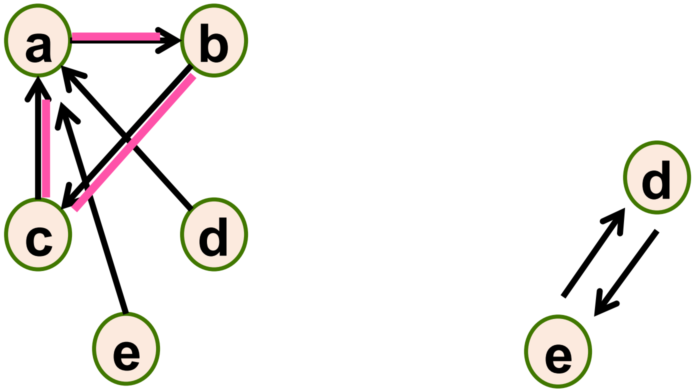


A w-chain

Gale’s Top Trading Cycles Algorithm

- Works for housing markets
  - needs adaptation to account for chains, etc. in kidney exchange
  - very natural, incredibly simple approach
- Each pair points to her favorite available kidney (could be her own)
  - must be at least one cycle (a *trading cycle*)
- Remove each cycle from the graph, allocating kidneys to the pairs pointing at them
- Repeat: each remaining pair points to favorite *remaining* kidney
- Terminate when each kidney assigned (termination assured)

a: *b > c > a > d > e*
b: *c > a > b > d > e*
c: *a > b > c > d > e*
d: *a > b > c > e > d*
e: *a > b > c > d > e*



lgorithmic and Other Issues

Information hiding by hospitals, manipulation

Scalability

- solving these problems is incredibly difficult (tens of thousands of pairs nationwide, now that exchange have become regional and national)
- a lot of research in computer science is making these problems solvable

Accommodating online (future) demand

- market is not static at all!
- there are arrivals and departures:
    - it better to use a current match, or wait for new donors and patients (which might improve overal outcome)?
    - an altruistic donor arrives: use now or wait for a better match, or ability to precipitate a longer chain, or…?
- requires even more difficult “optimization and modeling”

Wrapping up first term

The term consisted of approximately

- 7 lectures on networks and social networks
- 6 lectures on basic game theory
- 6 lectures on auctions including mechanism design
- One guess lecture (no slides) by Tyrone on a pricing problem
- 2 lectures on stable matchings and kidney exchange

Next term will be more focused on social network phenomena

Feedback: Comments now or by email or TAs about

- Course topics; what topics are not well understood?
- Workload; how much time do you estimate the course takes/week
- Grading; fair, timely?