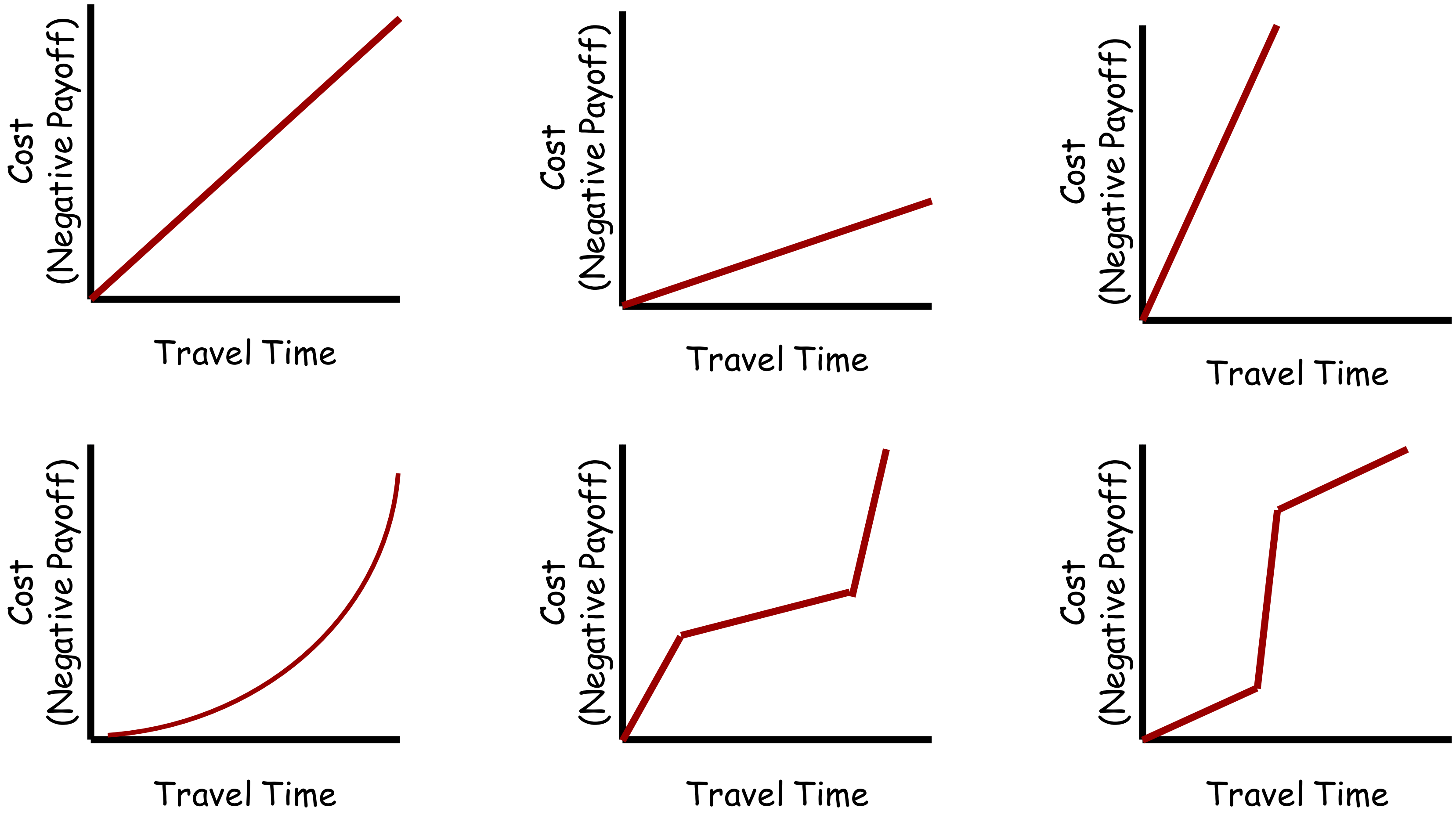
CSC200: Lecture 13

- Today
  - Transition to mechanism design
  - Auctions (Ch.9)
- Next few lectures
  - Auctions: Chapter 9 (plus additional topics)
  - Matching markets: Chapter10 (to be followed later by auctions for online advertising on search engines)
- Announcements
  - I have uncollected Quiz 1, Quiz 2 and Assignment 1. As I mentioned last time, I will only bring uncollected assignments once to the following class.
  - Quiz 3 on Friday, November 13; topic is game theory
  - Plan to have part or all of Assignment 2 available by weekend.
  - No class next Monday (Nov 9) due to "fall break".

Games with Unknown Structure

- Consider imposing socially optimal solution in Braess
  - Must assign 500 lucky drivers to ACDB group, 3500 to ACB/ACB
- How to assign? As stated, can be completely random!
  - Every driver has same cost (negative payoff) for driving minutes
    - *So social welfare is maximized no matter who gets assigned where*
- Is this a valid assumption? Obviously, not:
  - Some people value their time (out of the car) more than others
  - Value of commute time may not be linear:
    - e.g., once I'm in my car, long commute doesn't bother me
    - e.g., any commute time under 70 minutes is OK (prefer less of course), but cost of 70+ minutes is horrible (can't drop my kid at daycare before 7:50, and must be at work by 9:00)
  - Wide variety of possible *utility or cost functions* for time

Some Possible Cost vs Time Functions



How would you optimize social welfare?

- Could ask everybody to tell you their value for time
  - Now allocate drivers to routes so *sum of values* is maximized
- Why won't this work?
  - Everyone prefers lower times, so why not lie? Claim your time is more valuable than anyone else's
- Notice this is a new game
  - Drivers don't choose routes, they choose a *specific value of their time to declare*, then authorities assign them to routes
  - **IF** everybody told the truth, we'd be sure to get socially optimal outcome although sometimes can be difficult to compute OPT
  - But dominant strategy is to "maximally exaggerate"
  - So we get no useful info, and can't optimize social welfare!

Payments

- Suppose drivers must *pay a toll* for right to drive on C-D
  - Then they are expressing some information about the value of their time: time savings of taking C-D must be worth at least the price; i.e. each agent is *individually rational*.
  - Issues:
    - Time savings not predictable *a priori* (will emerge in NE)
    - How to set toll? If too low? If too high?
    - Assumes people willing to place monetary value on time
    - Doesn't account for budget constraints (rich vs. poor)
- Role of payments: forces people to express "some information" about the value of outcomes

Games with Incomplete Information

- So far our games have assumed everybody knows the moves of the game (their own and opponents') as well as payoffs (their own and opponents')
  - We call this the *complete information* assumption
  - Holds in normal form and extensive form games
- This is also wildly unrealistic in most settings
  - Knowing actions/strategies that others can take is not always reasonable
  - Knowing payoffs of other players is even less realistic
    - *e.g., value of short commute time to other drivers*
    - *e.g., value of revenue in specific area (A or B) to Telus*
  - These are often called *incomplete information games.* We'll focus on uncertainty in payoffs.

Mechanism Design

- Suppose policy maker wants to build new highway and ensure it is used to maximal benefit of society
- Can't determine maximal benefit unless you know individual preferences however
- Could you design a *game* that gets people (in equilibrium) to truthfully reveal their preferences for various outcomes?
  - e.g., the most they would be willing to pay to drive for *x* minutes
  - If so, you could assign outcomes that maximize social welfare

- *Mechanism design:* a branch of game theory that does just this. Informally, one designs games where preferences (or payoffs) are unknown (private information), so when players act "rationally" (in equilibrium) a socially desirable outcome emerges.
- Mechanism design is therefore a type of algorithm design where inputs are coming from self interested agents.

Auctions

- *Auctions* provide a canonical application of mechanism design.
- Simple example: I want to give or sell my old iPhone4 to the person in class who values it most (this will maximize social welfare).
  - Consider gov't auctioning wireless spectrum, logging, fishing, oil exploration rights, etc.
- How should I do this to insure that I am optimizing the social welfare (ignoring whatever revenue I may or may not get?)
- Lets do this now as a sealed bid auction.
- I am asking everyone to write on one piece of paper how much it is really worth to you and and on another paper what you are willing to bid for it (the bids are your actions). I collect (only see) the "bids".
- My goal is to give it to the person who values it the most. Can I use prices to make this happen?
- Note Tyrone was talking about many sellers and one buyer where each seller knows buyers value for their item. Now one seller and many buyers and seller does not know buyers value for item.

Auctions (selling my iPhone 4)

- Our simple auction example: I want to give (or sell) my old iPhone4 to the person. Want to give it to person who values it the most. How do I do this? Should I set a price and if so how?
- Procedure 1: Outcome is to give it to highest bidder at no cost.
  - Why wouldn't you all exaggerate (assuming you personally want a used iPhone)?
- Procedure 2: Outcome: give to highest bidder, but charge bid price
  - You won't exaggerate now? But won't you understate your true value?
- Procedure 3: Outcome: give to highest bidder, but charge the second-highest bid price
  - Now how will you bid?

Auctions

- Auctions widely used (to both sell, buy things)
  - our focus will be on *one-sided, sell-side (forward)* auctions: that is, we have a single seller, and multiple buyers
  - examples: rights to use public resources (timber, mineral, oil, wireless spectrum), fine art/collectibles, houses/property (Australia, UK, …), Ebay ($60B volume/yr), online ads (Google, Facebook, Microsoft, …), …
- Variations:
  - *multi-item auctions:* one seller, sells multiple items at once
    - e.g., wireless spectrum, online ads
    - interesting due to substitution, complementarities
  - *procurement (reverse) auctions:* one buyer, multiple sellers
    - common in business for dealing with suppliers
    - government contracts tendered this way
    - aim: purchase items from *cheapest* bidder (meeting requirements)
  - *double-sided auctions:* multiple sellers and buyers
    - stock markets a prime example, *matching* is the critical problem

Single-item Auctions (Sell-side/Forward)

- Assume seller with one item for sale
- Several different formats
  - **Ascending-bid (open-cry) auctions** (aka *English auctions*)
    - price rises over time, bidders drop out when price exceeds their "comfort level"; final bidder left wins item at last drop-out price
  - **Descending-bid (open-cry) auctions** (aka *Dutch auctions*)
    - price drops over time, bidders indicate willingness to buy when price drops to their "comfort level"; first bidder to indicate willingness to buy wins at that price
  - **First-price (sealed bid) auctions**
    - bidders submit "private" bids; highest bidder wins, pays price s/he bid
  - **Second-price (sealed bid) auctions**
    - bidders submit "private" bids; highest bidder wins, pays price bid by the *second-highest bidder*

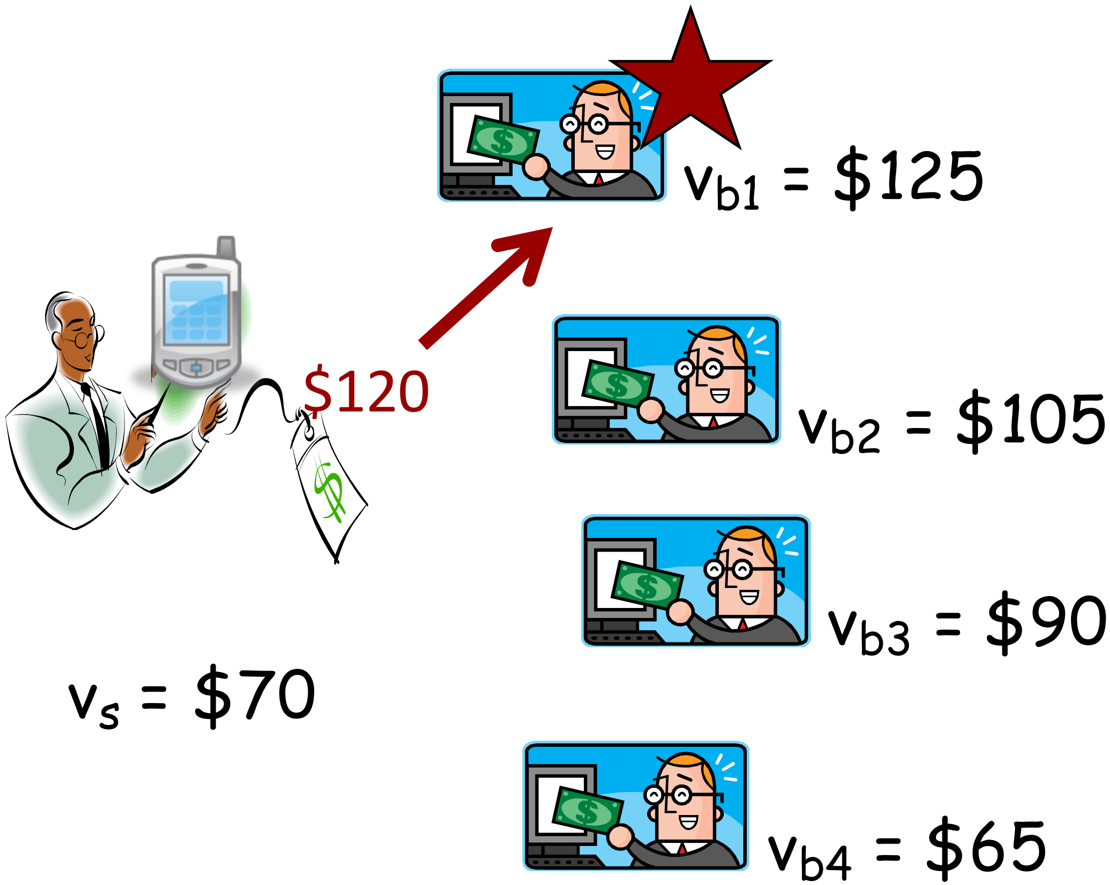
Why would seller use an auction?

- Let's assume the following:
  - seller *s* has a single item for sale
  - *s* values item at v_s (would rather keep than sell for less than v_s)
  - *s* is trying to maximize sale price (its own revenue)
  - set of potential buyers *B*, each *b* in *B* has value v_b for item
  - buyer values are *independent and private*
- *Independent, private valuations* means each buyer's value is "personal", does not depend on values of other buyers
  - consider items that will be consumed/used directly by *b*
- *Common values*
  - if *b* could resell item to another *b'*, then values are no longer independent or purely personal (*b* could speculate about value of item to others and buy it just for purpose of reselling)
  - values become correlated too if buyers are uncertain about the value of the item, and others have (different) private information about that value
    - *classic example: bidding for oil-drilling rights*

Why use an auction: what if values are known?

- If s knows values v_b of all buyers
  - s can offer item to b for price v_b (or a bit less), *if greater than* v_s
  - s would select the b^* with *highest value* v_{b^*}
    - alternatively, could just announce a *price* (a shade below) v_{b^*}
  - s extracts entire *surplus*: $v_{b^*} - v_s$
  - if b purchases at price v_b then net benefit or gain to b is zero
  - if b^* could bargain, could potentially reduce price
    - but s won't accept price below second-highest value: $v_{b(2)}$
- Notice that selling item this way maximizes social welfare
  - item goes to buyer (or stays with seller) that values it most
  - any price paid simply redistributes some of the surplus between the buyer and the seller (any price between v_{b^*} and v_s works)

Seller knows values: illustration



- Seller can set price anywhere between $105 and $125 (let's say $120)
- Buyer b1 with highest value will accept price; gain to b1: $5 (125-120); gain to s: $50 (120-70)
- Social welfare increases by $55 (125-70)
- Different price (or bargaining) splits the surplus between b1 and s (but s doesn't need to accept price below 105)

Why use an auction: values are unknown

- All sounds good, but *s* usually doesn’t know values
  - what if *s* sets price too high?
    - No transaction, lose social welfare (and revenue)
  - what if *s* sets price too low?
    - Transaction occurs, but item could go to bidder with lower value than *b**, lose social welfare (and revenue)
- Auction format is a way of discovering preferences/values
  - can be used to maximize social welfare, revenue, other objectives

Second-price Auction

- Bidders submit "sealed" bids; highest bidder wins, pays price bid by *second-highest bidder*
  - also known as *Vickery auction*
  - special case of *Groves mechanism*, *Vickery-Clarke-Groves (VCG) mechanism* (which we'll see in later chapters)
- 2nd-price seems weird but is quite remarkable
  - truthful bidding, i.e., bidding your true value, is a *dominant strategy*
- To see this, let's formulate it as a game

The Second-Price Auction Game

- *n* players (bidders)
- each player *k* has value v_k for item
  - assume v_k between [0,1] (for concreteness only)
- strategies/actions for player *k*: any bid b_k between [0,1]
- outcomes: player *k* wins, pays price *p* (2nd highest bid)
  - *more than n outcomes: outcome includes price paid by winner*
- payoff for player *k*:
  - if *k* loses: payoff is 0
  - if *k* wins, payoff depends on price *p*: payoff is $v_k - p$
- Notice: game differs in critical way from usual matrix form
  - *no player actually knows the payoffs of the other players*

Equilibrium: Second-Price Auction Game

- Even without knowing payoffs of others, it turns out that *bidding its true valuation is <u>dominant</u>* for every player *k*
  - strategy depends on valuation: but *k* selects b_k equal to v_k

- Let's see why deviation from *truthful bid* can't help (and could harm) *k*, regardless of what others do
- We'll consider two cases:
  a) if *k* wins with truthful bid $b_k = v_k$
  b) if *k* loses with truthful bid $b_k = v_k$

Equilibrium: Second-Price Auction Game

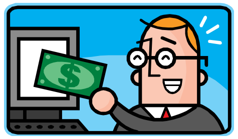
- Suppose k wins with truthful bid v_k
  - Notice k's payoff must be zero (if tied for 1st place) or positive
  - Now let's consider if k could have done better…
- Bidding b_k higher than v_k:
  - v_k already highest bid, so k still wins and still pays price p equal to second-highest bid $b_{(2)}$; so k is no better off
- Bidding b_k lower than v_k:
  - If b_k remains higher than second-highest bid $b_{(2)}$ then no change in winning status or price; so k is no better off
  - If b_k falls below second-highest bid $b_{(2)}$ then k now loses and is worse off (assuming v_k is great than $b_{(2)}$

Equilibrium: Second-Price Auction Game

- Suppose k loses with truthful bid v_k
  - Notice k's payoff must be zero and highest bid $b_{(1)} > v_k$
  - Now let's consider if k could have done better…
- Bidding b_k lower than v_k:
  - v_k already a losing bid, so k still loses and still gets payoff zero
- Bidding b_k higher than v_k:
  - If b_k remains lower than highest bid $b_{(1)}$, no change in winning status (k still loses)
  - If b_k is above highest bid $b_{(1)}$, k now wins, but pays price p equal to $b_{(1)} > v_k$ (payoff is negative since price is more than it's value)

- So a truthful bid is *dominant*: optimal no matter what others are bidding

Truthful Bidding in Second-Price Auction

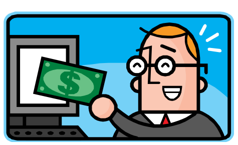


b_1 = \$125

v_2 = \$105

b_2 = ???

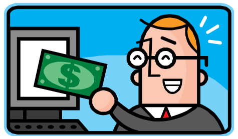
b_3 = \$90



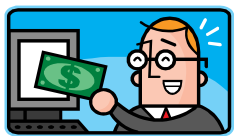
b_4 = \$65

- Consider actions of bidder 2
  - Ignore values of other bidders, consider only their bids. Their values don't impact outcome, only bids do.
- What if bidder 2 bids:
  - truthfully \$105?
    - loses (payoff 0)
  - too high: \$120
    - loses (payoff 0)
  - too high: \$130
    - wins (payoff -20)
  - too low: \$70
    - loses (payoff 0)

Truthful Bidding in Second-Price Auction



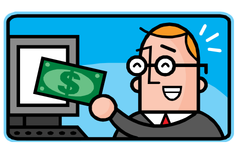
b_1 = \$95



v_2 = \$105

b_2 = ???

b_3 = \$90



b_4 = \$65

- Consider actions of bidder 2
  - Ignore values of other bidders, consider only their bids. Their values don't impact outcome, only bids do.
- What if bidder 2 bids:
  - truthfully \$105?
    - wins (payoff 10)
  - too high: \$120
    - wins (payoff 10)
  - too low: \$98
    - wins (payoff 10)
  - too low: \$90
    - loses (payoff 0)