CSC200: Lecture 8

- Today
  - Introducing game theory (Ch.6.1-6.4)
- Next Wednesday and next few lectures
  - game theory: Ch.6., Ch.8, Ch.9 (auctions)
- Announcements
  - Assignment 1 due on Friday, Oct.16 at start of class. I have clarified the wording on questions 1 and 4.
  - Quiz 2 : will take place Friday Oct. 23. I have posted a practice question and this is an indication of the scope of the quiz. You can discuss this practice question in this weeks tutorial but you will learn much more if you try it for yourself before tutorial.
  - Please go to where you took the quiz in case we are handing them back this Friday. Please note remarking policy.

Explaining Individual Behavior

- So far in Ch.3, 4:
  - discussed structural properties of networks (more next term!)
    - *neighborhood overlap, clustering coefficient, bridges, embeddedness,…*
  - examined individual processes that explains emergence of certain links/edges/relationships
    - *homophily (mutable/immutable), triadic/focal/membership closure, …*
- But we haven't tried to explain (in a precise manner) what drives individual agents to act the way they do, to respond to actions of those in their neighborhoods
  - why form a friendship with a friend of a friend?
  - why move when "people like me" move?
    - *Schelling model was our first attempt to move in this direction!*
- Game theory provides us with some tools to help explain individual behavior.

Interactions between agents

- A story (*the Schelling model in a microcosm at a personal level)*
  - *Suppose Alice and Bob want to live near each other, but prefer different neighborhoods (Alice: uptown, Bob: down)*
  - *Alice prefers uptown much more than downtown*
    - *would pay $400 more than her current rent to live uptown, nothing extra to live downtown (all else being equal in apartment quality)*
  - *Bob prefers to live downtown a bit more than uptown*
    - *would pay $300 more downtown, $200 more uptown*
  - *Both would pay an extra $200 to live near each other*
  - *Each gets a call from (different) landlords offering two apartments, same rent, one uptown and one downtown. Must decide immediately; can't communicate; know each other's preferences)*
- How should they act now? What would you predict?

A Game in Matrix Form

- We can model the relative benefits to each in a *matrix*
- **Key:** the value Alice associates with her action depends also on the action taken by Bob (and vice versa) *[*compare to Schelling model]*

Bob

Alice	Uptown	Downtown
Uptown	400 /	400 /
Downtown	0 /	0 /

Alice payoff for Nbrhood

+

	Uptown	Downtown
Uptown	200 /	0 /
Downtown	0 /	200 /

Alice payoff for collocation

=

	Uptown	Downtown
Uptown	600 /	400 /
Downtown	0 /	200 /

Alice payoff/

Bob

Alice	Uptown	Downtown
Uptown	/ 200	/ 300
Downtown	/ 200	/ 300

Bob payoff for Nbrhood

+

	Uptown	Downtown
Uptown	/ 200	/ 0
Downtown	/ 0	/ 200

Bob payoff for collocation

=

	Uptown	Downtown
Uptown	/ 400	/ 300
Downtown	/ 200	/ 500

/Bob payoff

A Game in Matrix Form

- We can model the relative benefits to each in a *matrix*
- **Key:** the value Alice associates with her action depends also on the action taken by Bob (and vice versa)

		Bob: Uptown	Bob: Downtown
Alice	Uptown	600 / 400	400 / 300
	Downtown	0 / 200	200 / 500

Alice payoff/Bob payoff

- Key points (two players)
  - *strategies* for one player listed along row, for other player along column
    - can be more than two strategies
  - *payoff* for *each* player for the joint strategy listed in each matrix cell
    - custom: row player/column player
  - model easily generalized to multiple players (but harder to draw)
- We'll discuss assumptions shortly

Predicting Behavior

		Bob: Uptown	Bob: Downtown
Alice	Uptown	600 / 400	400 / 300
	Downtown	0 / 200	200 / 500

- How will Alice behave?
  - If Bob moves uptown, she'd prefer to be uptown
  - If Bob moves downtown, she'd still prefer to be uptown
  - Thus, uptown is a *dominant strategy* for Alice (her preferred choice doesn't depend on Bob)
- How will Bob behave?
  - If Alice moves uptown, he'd prefer to be uptown
  - If Alice moves downtown, he'd prefer to be downtown
  - Thus, his preference depends on Alice's move
  - But he can predict Alice's move to uptown: so he should move uptown
- Our prediction is: *(Alice: uptown; Bob: uptown)*
  - This is an *equilibrium* of the game
  - Who "gets their way" in this game?

A Game in Matrix Form

- A *game in matrix (or normal) form* consists of:
  - a finite set of *players* (sometimes labeled *i, j, k, …*)
    - the agents (people, companies, etc.) that act in the game
  - for each player *i*, a set of *strategies* (sometimes denoted S_i)
    - the actions or options available to the agents
    - can be complex plans or courses of action (see extensive form)!
  - for each player *i*, a *payoff matrix* or *payoff function* (denoted P_i)
    - for each combination of strategies (one per player), indicates how good that combination is for player *i*
    - e.g., we write $P_2(s_1,s_2)$ or $P_{Alice}(downtown_{Alice},uptown_{Bob})$
    - in 2 or 3 player games, often written as a matrix or set of matrices

Underlying Assumptions

- Payoffs encode everything players care about
  - Not same as "selfishness" (e.g., altruism, empathy can be part of payoff)
  - *What if Bob's happiness mattered to Alice? Exercise: model it in the matrix*
  - *Is it easy for a person to actually assess precise numerical scores/payoffs?*
- Players are self-interested
  - Choose actions to *maximize their own payoff* *(*again, not = selfish)*
- Players all know the entire structure of the game
  - Know all players, all available strategies, all payoffs (not just your own)
  - Roughly, this allows each player ("you") to make some predictions of how other players will play (based on their predictions of how "you" will play, based on your predictions of how they will play, …)
  - A technical condition we need not worry about
  - We will see variants where players are uncertain about payoffs of other players (e.g., auctions), but still assume "common knowledge"

Another Game

- Alice and Bob, but with a slight variation in payoffs
  - Alice: uptwn 400, downtwn 100; Bob: uptwn 100, downtwn 400
  - Both: 200 bonus for being near each other
- Alice and Bob both have dominant strategies
  - Alice: if Bob U, U (600) > D (100); if Bob D, U (400) > D (300)
  - Bob: if Alice D, D (600) > U (100); if Alice U, D (400) > U (300)
- They will live apart: *(Alice: uptown; Bob: downtown)*

		Bob	
		Uptown	Downtown
Alice	Uptown	600 / 300	400 / 400
	Downtown	100 / 100	300 / 600

Another Game: An Unappealing Outcome

- Alice and her friends like to hang out with Bob and his friends (both have a large group of friends)
- There are two clubs
  - Club 1: great place if a big crowd, lots of friends; otherwise horrible
  - Club 2: exciting, but tiny: If too crowded (e.g., if both groups show up), not so great
  - payoffs: see payoff matrix
- Alice and Bob (their groups) *both* have dominant strategies
  - no matter what the other does, going to Club 2 is better
- Equilibrium *(Alice: Club2; Bob: Club2)* has payoff (4,4) that is *worse for both of them* than the payoff (6,6) if they both go to Club 1

		Bob	
		Club 1	Club 2
Alice	Club 1	6 / 6	2 / 7
	Club 2	7 / 2	4 / 4