Week 3: Introduction to Proofs

CSC165 L5102
Instructor: Felipe V.-H.

Mondays 6-8: GB119 (here)
Wednesdays 6-8: LM159
(not what it says on acorn, not where we were last week)
Exceptions: February 12, 26 (George Ignatieff Theatre)
We'll make announcements for all of the changes.

Problem Set 1 due Friday, January 24 by 4 PM
Extra office hours have been posted.

Sets, cartesian product x, functions
predicates, predicate logic

Sets: \[ \mathcal{E} \times \mathbb{N} : 2 \times 3 \]

Cartesian product:
\[ \mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} \]
is a set of
ordered triplets \((a, b, c)\) \(a, b, c \in \mathbb{R}\)
\[ \{1, 2, 3 \times \{3, 4\} = \{1, 4\}, \{1, 4\} \cup \{2, 3\} \]
\[ \{7, 4\} \]

\[ S^n = \mathcal{E} \times \mathcal{E} \times \mathcal{E} \times \mathcal{E} \times \mathcal{E} \]
\[ S = \mathcal{E} \times \mathcal{E} \quad n = 3 \]

A function \( f : \mathbb{R} \rightarrow \mathbb{R} \).
\[ \mathcal{G} f \in \mathcal{D} \times \mathbb{R} = \mathbb{R}^2 \]
\((0, 1) \in \mathbb{R}^2 \) \(\iff f(0) = 1 \)
\((0, 2) \in \mathbb{R}^2 \) \(\iff f(0) = 2 \)
\(D \Rightarrow \mathbb{R} \)
Each element of the domain appears at most once.

\[ f(x) = 2x + 1 \]

\[ f = \{(0,1), (1,3)\} \ldots \]

\[ f(x) = \frac{1}{x} \quad 0 \neq 0 \]

\[ f(x, y) = z \quad \mathbb{R}^2 \rightarrow \mathbb{R} \]

\[ f(x) = y, z \quad \mathbb{R} \rightarrow \mathbb{R}^2 \]

\[ f(x) = 2x, -2x \]

**Predicates are functions.**

where \text{Range} = \{T, F\}

\[ \forall x \in U, \text{Attends}(x) \text{ predicate} \]

\[ \text{Attends}(y) : "y \text{ attends lectures}" \]

\[ \begin{array}{c|c|c|c}
    x & z & 0 & 0 \\
\end{array} \]

**Predicate logic:**

Symbols, Rules, Meanings

\[ S = \forall x \in U, \text{Attends}(x) = \exists F \exists \]

Statement in predicate logic.
statement in predicate logic.

Proofs:
- show something (a program) is true
- express ideas mathematically
- prove something we know

Proofs are communication.

\[
\exists n \in \mathbb{R}; \; 1.5n - 4 \geq 3 \land n \geq 20 \\
\exists n \in \mathbb{R}; \; n > 20 \implies 1.5n - 4 \geq 3 \\
\forall n \in \mathbb{R}; \; n > 20 \implies 1.5n - 4 \geq 3 \\
\forall n \in \mathbb{R}; \; n > 20 \land 1.5n - 4 \geq 3
\]

Let \( n = 22 \), show \( 22 \geq 20 \land 1.5(22) - 4 \geq 3 \)

\[
\begin{align*}
22 & \geq 20 \text{ is True.} \\
33 - 4 & \geq 3 \\
29 & \geq 3 \text{ True.}
\end{align*}
\]

\[\therefore \exists n \in \mathbb{R}, n > 20 \land 1.5n - 4 \geq 3\]
1. Statement (English)
2. Translate in P.L.
3. Discussion (rough work)
4. Write the formal proof. Hand in

> 3 parts

Header: Introducing variables

assumptions

Proof body: deductions

- definitions
- axioms
- previous deductions
- external facts

Translations

Prove $\forall x \in \mathbb{Z}, x | x + 5 \Rightarrow x | 5$

Then generalize it!

$x | ax + b \Rightarrow x | b$

The proofs will be very similar.