0/H:  Tom  TR 3'-5'  BA2230
        HelpCentre  M-F 4-6  BA2230
        165 TAs to be announced.

Markus - check it out.

Steps in proof process.
1. Identify
2. Translate to predicate logic
3. Informal discussion

\[ d \mid n: \exists k \in \mathbb{Z}, \ n = k \cdot d \],

where \( d, n \in \mathbb{Z} \).

Statement:
Prove that for all integers \( x \),
if \( x \) divides \( (x+5) \) then
\( x \) divides 5.
0. Prove it. 2. Identify variable \( x \in \mathbb{Z} \).

\[ \forall x \in \mathbb{Z}, \ x \mid (x+5) \Rightarrow x \mid 5 \]

Using predicate definition:

\[ \forall x \in \mathbb{Z}, \ (\exists k \in \mathbb{Z}, \ x+5=k \cdot x) \Rightarrow (\exists k_2 \in \mathbb{Z}, \ 5=k_2 \cdot x) \]

3. Discuss: Try a few values of \( x \) since

Start about all \( x \in \mathbb{Z} \).

Let \( x = 5 \). Then \( x+5 = 5+5 \)

\[ = 10 \]

\[ = 2 \cdot 5 \]

\[ = k \cdot x \text{ for } k = 2 \]

So \( x \mid (x+5) \)

LHS \( \Rightarrow \) True

\[ \Rightarrow \text{true requires } \text{RHS} \Rightarrow \text{also be True} \]

Want to show \( x \mid 5 \). True since

\[ 5 = 1 \cdot 5 \]
Try $x = 10$

$x + 5 = 10 + 5$

$= 15$

Does $10 \mid 15$? No.

So $x \mid (x+5)$ is False

$x \mid (x+5) \Rightarrow x \mid 5$ is vacuously True

Try $x = 1$

$x + 5 = 1 + 5$

$= 6$

$= 6 \cdot 1$

$= k \cdot x$ for $k = 6$

So $x \mid (x+5)$

So LHS $\Rightarrow$ is True

RHS?

Does $1 \mid 5$? Yes since $5 = 5 \cdot 1$

$= k \cdot 1$

for $k = 5$. 

If LHS $\Rightarrow$ is False, $\Rightarrow$ is vac. True
if LHS of $\Rightarrow$ is True

then $\exists \, k \in \mathbb{Z}, \ x + 5 = k \cdot x$

To show RHS of $\Rightarrow$ is True

we need to find $k'$, such that

$5 = k' \cdot x$

Know

$x + 5 = k \cdot x$

$5 = k \cdot x - x$

$= (k-1) \cdot x$

$\therefore \exists \, k' \in \mathbb{Z}, \ 5 = k' \cdot x$

Proof

Let $x \in \mathbb{Z}$ be an arbitrary integer and assume $x \mid (x+5)$.

Then $\exists \, k \in \mathbb{Z}, \ (x+5) = k \cdot x$

Then $5 = k \cdot x - x$

$= (k-1) \cdot x$

Let $k' = k-1$. Then $5 = k' \cdot x$

and $\exists \, k' \in \mathbb{Z}, \ 5 = k' \cdot x$. That is $x \mid 5$.

Try to make statement more general.

$5 \rightarrow d \in \mathbb{Z}$
Consider
\[ \forall d \in \mathbb{Z}, \forall x \in \mathbb{Z}, x \mid (x+d) \Rightarrow x \mid d \]

Proof: as above, replace \( 5 \) by \( d \)

but start with let \( d \in \mathbb{Z} \), and ...

Prime numbers:

A natural number \( p \) is prime when it is greater than 1 and the only natural numbers that divide it are 1 and itself \((p)\).

That is:

\[
\text{Prime}(p): \quad p > 1 \land (\forall d \in \mathbb{N}, d \mid p \Rightarrow (d=1 \lor d=p))
\]

where \( p \in \mathbb{N} \)

Now consider:

\[
\forall p \in \mathbb{N}, \forall x \in \mathbb{N}, (\text{Prime}(p) \land x \mid (x+p)) \Rightarrow (x=1 \lor x=p)
\]

familiar from earlier
1. True (2) given

3. Suppose \( p, x \in \mathbb{N} \). Assume \( \text{Prime}(p) \) and also \( x \mid (x+p) \)

(to make \( \Rightarrow \) by path. True)

want to show \( x = 1 \) or \( x = p \) is True

We know from earlier that

\[ x \mid (x+p) \Rightarrow x \mid p \]

Also \( p \) is Prime

\[ \Rightarrow p > 1 \]

\[ \forall d \in \mathbb{Z}, d \mid p \Rightarrow (d = 1 \lor d = p) \]

RHS \( \Rightarrow \)

4. Proof.

Let \( p, x \in \mathbb{N} \) and assume \( p \) is prime and that \( x \mid (x+p) \).

Since \( \forall d \in \mathbb{Z}, \forall x \in \mathbb{N}, x \mid (x+d) \Rightarrow x \mid d \)

we can conclude \( x \mid p \)

Since \( p \) is prime, its only divisors
are 1 or p, and therefore we can conclude that $x=1$ or $x=p$.