Consider: \[ p \lor q \Rightarrow r \]

Interpret as: \[ (p \lor q) \Rightarrow r \]

or \[ p \lor (q \Rightarrow r) \]

Related: \[ a + b \times c \]

as \[ (a + b) \times c \] or \[ a + (b \times c) \]

precedence rule for arithmetic: \( \times \) before \(+\)

Precedence rules for logical operators:

- highest (apply first) \( \Rightarrow \)
- then \( \land, \lor \) (left to right)
- then \( \Rightarrow, \Leftrightarrow \)
- then \( \land, \lor \)

So interpret \( \circ \) as \( (p \lor q) \Rightarrow r \).

When in doubt, use \( (\ ) \)

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Ch 2 An Introduction to Proofs
defn: A proof is an argument that shows that a statement is True.

A disproof is False.

good proof: are concise
  complete (no details left out)
  presented in a connected order.

Proof Recipe/Procedure:

1. Identify: Prove or disprove?

2. Translate the statement to predicate logic

   Doing so: may help see how to prove it
   any possible
   forces you to resolve ambiguity in statement.

3. Informally write down your observations.
   express intuition about problem.

4. Write a formal proof that properly express your argument
Consider "Some power of two is greater than 1000."

1. **Suspect** True since powers of two grow to $\infty$ ($\therefore$ greater than 1000)

2. *What is the variable?*
   - The powers of two - call it $n$
   - All powers of two have form $2^n$
   - Some $\rightarrow$ existential

$$\exists n \in \mathbb{Z}, \quad 2^n > 1000.$$  

or

$$\exists n \in \mathbb{Z}, \quad P(n)$$

**$P(n)$:** "$2^n > 1000$", where $n \in \mathbb{Z}$.

Statement type: An existentially quantified simple predicate.

3. Know $2^n > 1$

<table>
<thead>
<tr>
<th>$n$</th>
<th>$2^n$</th>
<th>$2^n &gt; 1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>256</td>
<td>F</td>
</tr>
<tr>
<td>9</td>
<td>512</td>
<td>F</td>
</tr>
<tr>
<td>10</td>
<td>1024</td>
<td>T</td>
</tr>
</tbody>
</table>

**Since $n < 1$** give $2^n < 1$.
Proof.

Let $n = 10$.

Then $2^n$ is a power of two.

and $2^n = 2^{10}$

$= 1024$

$> 1000$.

Hence $\exists n \in \mathbb{Z}$ such that $2^n > 1000$.

$\square$

Summary: Fundamental Structure of an Existence Proof

Given statement to prove: $\exists x \in \mathbb{D}, P(x)$

Proof looks like:

- **header**

  [ ] Let $x = __$

- **body**

  [ ] Prove that $P(\_)$ is True.

Note: the $\_\_$ can be a concrete value or an expression that evaluates to a concrete value. You get to choose the value.
example: "Every real number $n$ bigger than 20 satisfies the inequality $1.5n - 4 \geq 3."$

1. Suspect True since LHS represents a line with positive slope and RHS is constant.
   
   Also $n = 40$  \[1.5(40) - 4 = 56\]  \[\geq 3\]

2. Variable $n$
   
   Domain: could choose $\mathbb{R}^{>20}$
   
   but choose $\mathbb{R}$ and write logic quantifier: $\forall$
   
   To impose condition $n > 20$
   
   $\forall n \in \mathbb{R}$, $n > 20 \Rightarrow 1.5n - 4 \geq 3$

3. Assume $n > 20$
   
   $\therefore$
   
   $1.5n > 30$
   
   $1.5n - 4 \geq 3$
   
   $1.5n - 4 > 26$
   
   $\geq 3$

   Write it up.

4. Proof.
   
   Let $n$ be an arbitrary real number and assume $n > 20$. 
Since \( n > 20 \)
1.5\( n \) > 30
1.5\( n \) - 4 > 26
1.5\( n \) - 4 > 3

Hence \( \forall n \in \mathbb{R}, \ n > 20 \Rightarrow 1.5n - 4 > 3 \). \( \square \)

**Summary.** Fundamental Structure of a Universal Proof

**Given statement:** \( \forall x \in D, R(x) \)

**Proof looks like:**

Let \( x \) be an arbitrary element of \( D \).

**Proof that \( R(x) \) is True.** \( \square \)

**Restriction on domain**

\( \forall x \in D, \ P(x) \Rightarrow Q(x) \)

**Proof looks like:**

Let \( x \) be an arbitrary element of \( D \) and assume \( P(x) \).

**Proof that \( Q(x) \) is True** \( \square \)

\[ P(x) \Rightarrow Q(x) \]

is True when \( P(x) \) is False
∀x∈D, P(x) = Q(x) \\
⇒ vac. true for x∈D \\
s.t. P(x) false \\

hard part: show ⇒ True \\
for x∈D s.t. P(x) True