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HelpCentre TAs 2 - 6 BA2230
165 TAs - will announce.

Prove More Statements.

Steps:
1. Identify
2. Translate to Predicate Logic
3. Informal discussion
4. Formal proof

divides predicate

d | n: "\exists k \epsilon \mathbb{Z}, n = k \cdot d", where d, n \epsilon \mathbb{Z}.

Statement to consider:
Prove that for all integers \( x \), if \( x \) divides \( (x+5) \) then \( x \) divides 5.

1. told to prove.
2. variable: \( x \epsilon \mathbb{Z} \).
   quantifier: \( \forall \)
∀ x ∈ ℤ, x | (x+5) \implies x | 5

(a universally quantified implication)

"unpack" predicate \mid def

∀ x ∈ ℤ, (\exists k_1, k_2 ∈ ℤ, (x+5) = k_1 \cdot x) \implies

(\exists k_2 ∈ ℤ, 5 = k_2 \cdot x)

③ discuss - try a few values of x

Let x = 5. Then x+5 = 5+5

= 10

= 2 \cdot 5

= 2 \cdot x

= k_1 \cdot x \text{ with } k_1 = 2

so \ x \mid (x+5)

LHS of implication True

Consider RHS of implication.

\ x \mid 5 \text{ is true}

5 \mid 5 \text{ yes since } 5 = 1 \cdot 5

RHS of implication True. 
Implication True.

Let $x = 10$.

\[
x + 5 = 10 + 5 = 15
\]

Does $10 \mid 15$? No

\[
x \mid (x+5) \Rightarrow x \mid 5 \quad \text{is vacuously True.}
\]

Let $x = 1$.

\[
x + 5 = 1 + 5 = 6
\]

Does $1 \mid 6$? Yes, $6 = k \cdot 1$ for $k = 6$.

$LHS \Rightarrow \text{is True.}$

\[
\text{RHS: } \text{Does } 1 \mid 5 \text{? Yes, } 5 = k \cdot 1 \text{ for } k = 5
\]

Hard part: case $b \neq x$ that make $LHS \Rightarrow$ True.

\[
\text{If } LHS \Rightarrow \text{is True, then } \exists k \in \mathbb{Z}, \quad x + 5 = k \cdot x
\]

To show that $\Rightarrow$ is True, need to show that $\text{RHS} \Rightarrow$ is True.

Want to find a $k'$ s.t. $5 = k'x$. 

to show this.

Start with what we know:

\[ k \cdot x \]

so \[ S = k x - x \]

\[ = (k-1)x \]

\[ \exists k' \in \mathbb{Z}, \ S = k'x \]

\[ \text{take } \ k' = k-1 \]

Proof.

Let \( k' \in \mathbb{Z} \) be an arbitrary integer and assume \( x \mid (x+5) \).

Then \( \exists k' \in \mathbb{Z}, \ (x+5) = k'x \)

\[ S = kx - x \]

\[ = (k-1)x \]

Let \( k' = k-1 \). Then \( S = k'x \)

and \( \exists k' \in \mathbb{Z}, \ S = k'x \). That is \( x \mid S \)

\[ \square \]

Try to think of generalizations of statements.

Replace \( S \) by \( d \in \mathbb{Z} \).

Consider: \( \forall d \in \mathbb{Z}, \forall x \in \mathbb{Z}, \ x \mid (x+d) \Rightarrow x \mid d \)

to prove: Let \( d \in \mathbb{Z} \) \( \ldots \) (rest of problem, replace \( S \) by \( d \)
A harder example:

A natural number $p$ is prime when it is greater than 1 and the only natural numbers that divide it are 1 and itself ($p$).

$$\text{Prime}(p) : \quad p > 1 \land \left( \forall d \in \mathbb{N}, \, d \mid p \Rightarrow (d = 1 \lor d = p) \right)$$

where $p \in \mathbb{N}$

Consider:

$$\forall p \in \mathbb{N}, \forall x \in \mathbb{N}, \left( (\text{Prime}(p) \land x \mid (x+p)) \Rightarrow (x = 1 \lor x = p) \right)$$

1. True
2. Given

3. Discuss: Suppose $p, x \in \mathbb{N}$.

Assume $\text{Prime}(p)$ and $x \mid (x+p)$

(to make LHS of $=$ True)

Want to show it follows that $x = 1 \lor x = p$

Know from test example. $x \mid (x+p) \Rightarrow x \mid p$
by the def. of prime, know \( p > 1 \) and \( \forall \text{divisor } d \mid Bp, d \text{ is } 1 \text{ or } p \) \( \rightarrow \) what we want.

(4) **Proof.** Let \( p, x \in \mathbb{Z} \) and assume \( \text{p is prime and that } x \mid (x+p) \).

Since \( \forall d \in \mathbb{Z}, \forall x \in \mathbb{Z}, x \mid (x+d) \Rightarrow x \mid d \).

we can conclude \( x \mid p \).

Since \( p \) is prime its only divisors are 1 or \( p \), and we can therefore conclude that \( x=1 \) or \( x=p \).

\( \square \)

**note:** use previously proven results

- unpack def to see how to use.