Consider: \( p \lor q \Rightarrow r \)  

interpreted as: \( (p \lor q) \Rightarrow r \)
\[ p \lor (q \Rightarrow r) \] ?

\[ a + b \times c \]
precedence: \( * \) before \( + \)

**Logic precedence rules:**

- highest (do first)  \( \neg \)
- then  \( \land, \lor \) (left to right)
- then  \( \Rightarrow, \equiv \)
- then  \( \forall, \exists \)

1. interpreted as: \( (p \lor q) \Rightarrow r \)

Ch 2  An Introduction to Proofs.

**Def.** A **proof** is an argument that shows that a statement is True.
A disproof is False.

Proof Recipe: process to follow.

1. Identify: proof or disproof.

2. Translate statement to predicate logic
   . form of statement guides proof approach
   . address any ambiguity.

3. Informally write down observations or intuition.

4. Write a formal proof that completely and concisely expresses your argument.

Consider: "Some power of two is greater than 1000."

1. Try to prove it since powers of 2 grow to infinity.

2. Variable: the power of 2 - call it n
   . n ∈ \(\mathbb{Z}\).
   . \(n \geq 2^n\)
   . all n? no, just some

\[\exists n \in \mathbb{Z}, \quad 2^n > 1000\]
or \( \exists n \in \mathbb{Z}, \ P(n) \)

where \( P(n): \ 2^n > 1000 \), where \( n \in \mathbb{Z} \).

\textbf{type:} An existentially quantified simple predicate

\( \exists \text{ know } \ n \geq 0 \text{ so } 2^n \geq 1 \)

\textbf{try a few values:} \( n = 8 \quad 2^8 = 256 \)
\( 9 \quad 2^9 = 512 \)
\( 10 \quad 2^{10} = 1024 \)

\( \textbf{Proof.} \)

Let \( n = 10 \).

Then \( 2^n \) is a power of 2 and
\[ 2^n = 2^{10} = 1024 > 1000. \]

Hence, \( \exists n \in \mathbb{Z}, 2^n > 1000. \)

\( \square \)

\textbf{Fundamental Structure of an Existential Proof.}

\textbf{Given statement:} \( \exists x \in D, P(x) \).

\textbf{Proof looks like:}
Let \( x = \_ \) (a concrete value)

Prove that \( P(\_) \) is True.

Note: you get to choose what goes in \( \_ \).

Consider "Every real number \( n \) bigger than 20 satisfies the inequality \( 1.5n - 4 \geq 3 \)."

1. Do you think True?
   - Yes: LHS: \( e \) graph of line, \( \Theta \) slope so goes to \( +\infty \)
   - RHS is constant

   \( n = 20 \), \( 26 \geq 3 \) \( \checkmark \)

2. Translate: variable \( n \) domain \( \mathbb{R} \) or \( \mathbb{R}^{>20} \)

   Use the largest domain and logic to restrict.

   \( \forall n \in \mathbb{R}, \ n^{>20} \Rightarrow 1.5n - 4 \geq 3 \).

3. Since \( n \) appears on both sides of
   write what we assume true

   \( n^{>20} \)
and try to show conclusion \(1.5n - 4 \geq 3\) is\(\text{True}\).

\[
\begin{align*}
\text{header} & \quad \text{variables} & \quad \text{assumptions} \\
\text{body} & \quad \text{argument} & \quad \text{leading} & \quad \text{to conclusion} \\
\end{align*}
\]

\[
\begin{align*}
n & > 20 \\
1.5n & > 30 \\
1.5n - 4 & > 26 \\
1.5n - 4 & > 3
\end{align*}
\]

Simple predicate \(R(n)\)

Fundamental Structure of a Universal Proof:

Given statement: \(\forall n \in D, R(n)\)
Proof look like:

Let $n$ be an arbitrary element of $D$.

Proof that $R(n)$ is True.

\[
\forall n \in D, \quad P(n) \Rightarrow Q(n)
\]

if restriction on domain:

\[
\text{True for } n \text{ for which } \neg P(n)
\]

structure becomes:

Let $n$ be an arbitrary element of $D$ and assume $P(n)$.

Prove that $Q(n)$ is True.

\[
\text{e.g. } n = 4, \quad 1.5n - 4 > 3 \text{ is False \quad but} \quad n > 20 \Rightarrow 1.5n - 4 \geq 3 \text{ is True}
\]