Ps 2. Q3 period $p \in \mathbb{R}^+$

$\forall x \in \mathbb{R}, \quad f(x+p) = \ldots$

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Induction

\[ P(m) \land \forall k \in \mathbb{N}, \quad k \geq m \land P(k) \implies P(k+1) \]

Sometimes need:

\[ P(m) \land P(m+1) \land P(m+2) \land \ldots \land P(k) \]

\[ \text{vsSimple to show } P(k+1). \]

Strong Induction

Example: Every integer $n > 2$ can be expressed as a product of one or more prime numbers.

Predicate: $P(n)$: "$n$ can be expressed as the product of one or more prime numbers" where $n \in \mathbb{Z}$.

\[ \exists m \in \mathbb{N}, \quad m > 1 \quad \land \quad \exists p_1, p_2, \ldots, p_m \quad \land \quad \text{Prime}(p_j) \quad \land \quad n = \prod_{j=1}^{m} p_j \]

Translate: \[ \forall n \in \mathbb{Z}, \quad n > 2 \implies P(n) \]
base case

Let \( n = 2 \).

\( P(2) \) is True since 2 is a prime number and so can be expressed as a product of one or more prime numbers.

inductive step: Let \( k \in \mathbb{Z} \) and assume \( k \geq 2 \)

and assume \( \forall j \in \mathbb{Z}, \ 2 \leq j \leq k \Rightarrow P(j) \)

i.e. for \( 2 \leq j \leq k \), \( j \) can be expressed as a product of prime numbers.

We want to prove that \( k+1 \) can be expressed as a product of prime numbers.

Either \( k+1 \in \mathbb{Z} \) is prime or it isn’t.

case: \( k+1 \) is prime

Then \( P(k+1) \) is True since \( k+1 \) expressed as itself.

case: \( k+1 \) is not prime

This \( k+1 = a \cdot b \) for some
positive integers \( a, b \) with \( 1 < a, b < k+1 \)

\[ \text{i.e. } \frac{1}{k} \]

And since \( 2 \leq a \leq k \) and \( 2 \leq b \leq k \), by the induction hypothesis, both \( a \) and \( b \) can be written as a product of prime numbers, and thus so can \( a \cdot b \) and \( P(k+1) \) is true.

Why not simple induction?

Since for \( k \geq 2 \), \( k \) is not a factor of \( k+1 \). Need strong induction to get correct factors with desired properties.

Ch 4: Representation of Natural Numbers

165

represents:

\[ 1 \times 100 + 6 \times 10^1 + 5 \times 1 \]

\[ = 1 \times 10^2 + 6 \times 10^1 + 5 \times 10^0 \]

Powers of 10: decimal/base 10 representation

Digits: multiples of powers of 10
\[ n \in \{0, 1, 2, \ldots, 9\} \]

- let \( d_i \) represent the multiple of \( 10^i \)

\[ d_0 = 5, \ d_1 = 6, \ d_2 = 1 \]

\[ 165 = \sum_{i=0}^{2} d_i \cdot 10^i \]

(165) \(_{10} \) decimal/base 10 representation

other:
- older IBM hexadecimal base 16
- Honeywell octal 8
- Setun ternary 3

In general, base \( \beta \),

\[ n = t_0 \cdot \beta^0 + t_1 \cdot \beta^1 + t_2 \cdot \beta^2 + \ldots \]

\[ = \sum_{i=0}^{\infty} t_i \beta^i \]

where \( |\beta| > 1 \), \( \beta \in \mathbb{N} \)

\[ \forall i \in \mathbb{N}, \ 0 \leq t_i < \beta \]

since \( n \) finite, \( \exists k \in \mathbb{N}, \ i > k \Rightarrow t_i = 0 \)
\( b = 10 \) (decimal), \( n \) called digits
\( \beta = 2 \) (binary), \( t \) called bits

**binary:** two choices \( b_i \in \{0,1\} \)

Look like: \( (\ldots 0010100101)_2 \)

Easy to find decimal form for this binary number.

\[
= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 \\
+ 1 \times 2^5 + 0 \times 2^6 + 1 \times 2^7 + 0 \times 2^8 \\
+ 0 \times 2^9 + \ldots \\
= 1 + 0 + 4 + 0 + 0 + 32 + 0 + 128 \\
= (165)_{10}
\]

Python str S containing '0', '1' s

d = 0
for i in range(len(S)):
    d = d + int(S[-(i+1)]) * 2**i
trickier to go from decimal to binary:

if \( n \) is odd: what is the rightmost bit? (least significant bit)

\[
\begin{align*}
\text{even} & : 0 \\
1 & \\
\text{odd} & : b = n \mod 2
\end{align*}
\]

if \( n \mod 2 = 0 \):

\[ b = 0 \]

else:

\[ b = 1 \]

gives the multiplier of \( 2^0 \).

\((n - b)\) must be even.

\[ \text{lsb in } (n - b) \div 2 \text{ will be multiplier of } 2^1 \text{ in } n. \]

Repeat this until \( \div 2 \) leaves nothing.
given \( n \in \mathbb{N} \), decimal

\[ s = '' \]

while \( n > 0 \):
\[ s = \text{str}(n^202) + s \]
\[ n = n / 2 \]

# S contains the binary representation of n

Trace: \( n = 165 \)

\[
\begin{array}{c|c}
\hline
\frac{n}{165} & s \\
\hline
110 & \text{"} \text{"} \\
\hline
\end{array}
\]

\( n^202 \text{ is } 1 \)

\( \text{\texttt{1}} \)

\( n / 2 = \text{floor}(n / 2) \)

82 when \( \text{\texttt{01}} \)

41

\( \text{\texttt{101}} \)

20

\( \text{\texttt{0 101}} \)

10

\( \text{\texttt{0 0101}} \)

5

\( \text{\texttt{1 00101}} \)

2

\( \text{\texttt{0 100101}} \)

1

\( \text{\texttt{10100101}} \)