CSC148 - Running time efficiency: Lists, Stacks, and Queues

We have now seen that Python lists have the following running times for key operations:

- Accessing or assigning to any element by index takes constant time.
- Inserting or removing an item at a given index takes time proportional to the number of items after the index.

1. Answer the following questions to make sure you understand the key concepts before moving on.
   
   (a) What do we mean by “constant time” above?

   (b) Suppose we have a list of length $n$. If we want to insert a new item at position $i$, how many list elements must be moved? (Be careful about off-by-one errors!)

   (c) Suppose we have a list of length $n$. If we want to remove the existing item at position $i$, how many list elements must be moved? Do not include the item being removed.

   (d) Suppose we have two lists: one of length 100, and one of length 1,000,000. Give an example of each of the following:

   (i) An operation that would be faster on the smaller list than the larger list.

   (ii) An operation that would take roughly the same amount of time on the two lists.

   (e) Suppose we implement a Queue using a Python list, where the front of the list represents the front of the queue. Which operation do we expect to take longer as the queue size increases: Queue.enqueue or Queue.dequeue?

2. Now let’s look at some code. Suppose we have two implementations of the Stack ADT: Stack1 has push and pop operations that take 1 step, regardless of stack size, while Stack2 has push and pop operations that take $n$ steps, where $n + 1$ is the number of items currently on the stack. (You might argue it’s “$n$ steps”, but that difference doesn’t matter here.)

   For each of the code snippets on the next page, calculate the number of steps taken in total by all push and pop operations that are performed by the code. Do each calculation twice: once assuming we use the Stack1 implementation, and once assuming we use the Stack2 implementation. Ignore all other operations for this exercise—you’re only counting steps for push and pop here.
(a)  # s starts as a stack of size n
    s.push(1)
    s.pop()

Stack1 | Stack2
--------|--------
        |        

(b)  # s starts as an empty stack
    for i in range(5):
        s.push(i)

Stack1 | Stack2
--------|--------
        |        

(c)  # s starts as an empty stack, k is a positive integer.
     # Calculate the number of steps in terms of k.
     # Hint: 1 + 2 + 3 + ... + k = k * (k + 1) / 2
     for i in range(k):
         s.push(i)

Stack1 | Stack2
--------|--------
        |        

(d)  # s1 starts as a stack of size n, and s2 starts as an empty stack
    while not s1.is_empty():
        s2.push(s1.pop())
    while not s2.is_empty():
        s1.push(s2.pop())

Stack1 | Stack2
--------|--------
        |        